# Multiple-Symbol Differential Sphere Decoding Aided Cooperative Differential Space-Time Spreading for the Asynchronous CDMA Uplink

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Abstract-In this paper, we propose a Cooperative Differential Space-Time Spreading (CDSTS) scheme employing multiple relays, which eliminates the demanding requirement of channel estimation both at the relays and at the destination. More explicitly, the source node employs differential encoding during the first transmission interval, and the multiple relays perform Differential Space-Time Spreading (DSTS) based Amplify-and-Forward (AF) relaying in the second transmission interval. Eliminating high-complexity channel estimation is particularly important at the light-weight shirt-pocket-sized mobile handset based relays, since it is unrealistic to estimate the associated mobile-to-mobile channels, which would also pose a security threat and impose a Doppler-dependent pilot overhead. Finally, it would be prone to a channel-estimation-induced performance degradation, which might be close to the 3 dB non-coherent performance penalty. Loosely Synchronized (LS) CDMA spreading codes were adopted for the asynchronous CDMA uplink for the sake of achieving a near-single-user performance using a low-complexity singleuser matched-filter detector. The potential performance degradation of the noncoherent receiver experienced in fast fading channels is mitigated by the proposed Multiple-Symbol Differential Sphere Decoding (MSDSD).

## I. INTRODUCTION

Space-Time Block Codes (STBCs) [1], [2] employing multiple antennas provide an effective means of mitigating the deleterious effects of channel fading. Inspired by STBCs, Space-Time Spreading (STS) was proposed in [3] in order to achieve both a multi-user support capability and a diversity gain in the context of Code Division Multiple Access (CDMA) systems. However, due to the size, cost or other hardware limitations, multiple antennas associated with insufficient element-spacing may experience spatially correlated fading, which erodes the diversity gain. To mitigate this problem, cooperative schemes were proposed in [4]–[6], where the singleelement Mobile Stations (MSs) may share their antennas to form a Virtual Antenna Array (VAA) and as a benefit, they typically experience spatially uncorrelated fading.

It is well recognized that coherently detected cooperative schemes require Channel State Information (CSI), which becomes hard to estimate, when the Source-Destination (SD) links, the Source-Relay (SR) links and the Relay-Destination (RD) links are rapidly changing due to the movement of the source and/or the relays. To avoid the potentially excessive complexity of coherent detection for distributed MIMO schemes, the family of Differential STBCs (DSTBCs) was proposed in [7], [8], and the corresponding Differential STS (DSTS) schemes were summarized in [9]. Against this background, an Amplify-and-Forward (AF) relaying aided Cooperative DSTS (CD-STS) scheme employing four relays designed for the asynchronous CDMA uplink was proposed in [10], in which Loosely Synchronized (LS) codes [11] were adopted for the sake of eliminating the Multi-User Interference (MUI) despite using a low-complexity matchedfilter-based single-user-receiver. In the absence of CSI estimation, the Conventional Differential Detection (CDD) generally suffers from a 3 dB performance penalty compared to its coherent counterpart. Furthermore, typically an irreducible error floor is formed when the Doppler frequency increases. To circumvent this problem, Multiple-Symbol Differential Decoding (MSDD) was proposed for noncoherent schemes in [12], [13] in order to mitigate the potential performance-erosion at high Doppler frequencies. As a further advance, Multiple-Symbol Differential Sphere Decoding (MSDD) was introduced in [14], [15] in order to limit the exponentially increasing complexity of MSDD, when increasing the detection window width.

Against this backcloth, the novel contribution of this paper is that we propose an MSDSD aided CDSTS for the asynchronous CDMA uplink, where neither CSI estimation nor symbol-level synchronization is required, yet, a good performance is guaranteed by the MSDSD. The system model of [10] is adopted, and a general model for CDSTS using multiple relays is presented. The MSDSD designed for cooperative AF relaying using DPSK [16] is further developed for the case of CDSTS.

This paper is organized as follows. A general model developed for CDSTS using multiple relays is presented in Section II, while the proposed MSDSD designed for the CDSTS is portrayed in Section III. Finally, our conclusions are offered in Section IV.

## **II. SYSTEM OVERVIEW**



Fig. 1. Block diagram of the CDSTS for the asynchronous CDMA uplink.

The block diagram of the proposed system is shown in Fig. 1. During the first T symbol periods, which is referred to as the broadcast interval, the K Source Nodes (SNs) spread their differentially encoded symbols by their user-specific signatures, and then transmit the CDMA-chips to both the Relay Nodes (RNs) and to the Destination Node (DN). Each user relies on M RNs, where the corresponding user's CDMA-chips are de-spread and then re-spread based on DSTS. During the following symbol period, which is also referred to as the cooperation interval, the DN receives the DSTSchips forwarded from the RNs. The DN de-spreads the CDMAchips and the DSTS-chips received from the SNs and from the RNs, respectively. Finally, MSDSD is invoked at the DN. In the proposed system, a frame-synchronized Time Division aided CDMA

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(TD-CDMA) model is adopted, which implies that no symbol-level synchronization is required.

In this paper, we focus our attention on real-valued BPSK/PAM signalling. In the proposed system, the signals received from the M RNs are faded by independent complex-valued Channel Impulse Responses (CIRs). Therefore the coherently-detected orthogonal complex-valued  $G_2$  structure [1] cannot be formed at the RNs. Instead, we derive a universal system model for the DSTS of Fig. 1 using M=2, 3 and 4 RNs.

The following notation is used throughout the paper. The number of RNs per user is denoted by M, which corresponds to the number of transmit antennas used for DSTS. The number of channel uses (symbol periods) and the number of transmitted symbols per DSTS block are denoted by T and Q, respectively, while the notation used for the different DSTS schemes is summarized in Table I. The total number of users (SNs) and RNs becomes K and  $N_R = KM^1$ , respectively. The total number of user-specific signatures required at the SNs and that of the unique DSTS spreading-sequences required at the RNs are K and KT, each of which has  $L_S$  and  $L_R$  chips, respectively. Moreover, N and  $N_w$  refer to the number of receive antennas and the MSDSD window length employed at the DN. Furthermore, we employ matrix representations throughout this paper for the convenience of block-wise processing in our MSDSD scheme.

TABLE I SUMMARY OF THE NOTATION FOR DSTS

DSTS using two transmit antennas	M = 2	T=2	Q = 2
DSTS using three transmit antennas	M = 3	T = 4	Q = 4
DSTS using four transmit antennas	M = 4	T = 4	Q = 4

A. Source Node

During the broadcast phase, the k-th user firstly maps its information bits  $b_k(q)$  to the BPSK/PAM symbols vector  $\mathbf{x}_k(i) = [x_k^1(i), \ldots, x_k^Q(i)]^T$ . The differential encoding process employed at the single-element-based SN is expressed as [9]:

$$\mathbf{s}_k(i) = \begin{cases} \mathbf{s}_k(0) & i = 1\\ \frac{1}{\theta} G_T(\mathbf{x}_k(i)) \cdot \mathbf{s}_k(i-1) & i > 1 \end{cases},$$
(1)

where the transmission matrix  $\mathbf{s}_k(i)$  has  $(T \times 1)$  elements, and the process commences with the all-one reference symbol matrix  $\mathbf{s}_k(0)$ . The normalization factor of  $\theta = || \mathbf{s}_k(i-1) ||$  becomes a constant of  $\sqrt{2}$  for BPSK signalling. In the case of higher-order multi-level modulation, this factor has to be estimated at the receiver [18]. The  $(T \times T)$ -element information matrix  $G_T(\mathbf{x}_k(i))$  represents the orthogonal STBC structure, which was summarized in [9] as:

$$G_{2}(\mathbf{x}_{k}(i)) = \begin{bmatrix} x_{k}^{1}(i) & x_{k}^{2}(i) \\ -x_{k}^{2}(i) & x_{k}^{1}(i) \end{bmatrix},$$

$$G_{4}(\mathbf{x}_{k}(i)) = \begin{bmatrix} x_{k}^{1}(i) & x_{k}^{2}(i) & x_{k}^{3}(i) & x_{k}^{4}(i) \\ -x_{k}^{2}(i) & x_{k}^{1}(i) & x_{k}^{4}(i) & -x_{k}^{3}(i) \\ -x_{k}^{3}(i) & -x_{k}^{4}(i) & x_{k}^{1}(i) & x_{k}^{2}(i) \\ -x_{k}^{4}(i) & x_{k}^{3}(i) & -x_{k}^{2}(i) & x_{k}^{1}(i) \end{bmatrix}.$$
(2)

Actually, the differential encoding operation of Eq. (1) is a variant of the DSTBC encoding process, which may be formulated as [9]:

$$\widetilde{G}_M(\mathbf{s}_k(i)) = \begin{cases} \widetilde{G}_M(\mathbf{s}_k(0)) & i = 1\\ \frac{1}{\theta} G_T(\mathbf{x}_k(i)) \cdot \widetilde{G}_M(\mathbf{s}_k(i-1)) & i > 1 \end{cases}, \quad (3)$$

<sup>1</sup>We also note that in practice the number of available RNs may not be as high as  $N_R = KM$ , but we set aside this resource allocation issue for our future research [17].

where the DSTBC transmission matrix  $\tilde{G}_M(\mathbf{s}_k(i))$  has  $(T \times M)$  elements. They may be summarized as:

$$\widetilde{G}_{2}(\mathbf{s}_{k}(i)) = \begin{bmatrix} s_{k}^{1}(i) & s_{k}^{2}(i) \\ s_{k}^{2}(i) & -s_{k}^{1}(i) \end{bmatrix}, 
\widetilde{G}_{4}(\mathbf{s}_{k}(i)) = \begin{bmatrix} s_{k}^{1}(i) & s_{k}^{2}(i) & s_{k}^{3}(i) & s_{k}^{4}(i) \\ s_{k}^{2}(i) & -s_{k}^{1}(i) & s_{k}^{4}(i) & -s_{k}^{3}(i) \\ s_{k}^{3}(i) & -s_{k}^{4}(i) & -s_{k}^{1}(i) & s_{k}^{2}(i) \\ s_{k}^{4}(i) & s_{k}^{3}(i) & -s_{k}^{2}(i) & -s_{k}^{1}(i) \end{bmatrix},$$
(4)

and  $\widetilde{G}_3(\mathbf{s}_k(i))$  is obtained by taking the first three columns of  $\widetilde{G}_4(\mathbf{s}_k(i))$ . Therefore, the differential encoding process of Eq. (1) is equivalent to the DSTBC encoding process, but only a single column of the transmission matrix of Eq. (3) is retained, because all MSs use a single antenna element.

For the K users supported by the system, the family of spreading codes  $\{\bar{\mathbf{c}}_k\}_{k=1}^K$ , which have  $L_S$  chips and a power of  $G_S$ , are employed by the SNs. The k-th user now spreads every symbol with the aid of its unique signature, yielding:

$$\mathbf{y}_k(i) = \sqrt{\frac{P_S}{G_S}} (\mathbf{I}_{\mathbf{T}} \otimes \bar{\mathbf{c}}_k) \cdot \mathbf{s}_k(i),$$
(5)

where  $\otimes$  denotes the Kronecker product, and the resultant chip-level transmission matrix  $\mathbf{y}_k(i)$  has  $(TL_S \times 1)$  elements, which implies that each transmission block requires  $TL_S$  chip periods, as well as a single transmit antenna. Furthermore,  $P_S$  represents the source's transmission power, while the transmit power of the relays is denoted by  $P_R$ . For the sake of appropriate power normalization in our AF cooperative system, we have a total transmit power of  $P_S + P_R = 1$ .

## B. Relay Node

During the broadcast interval, the *m*-th RN receives the superimposed  $L_S$ -chip CDMA signals from all the K users. In the asynchronous CDMA uplink, all signals arrive with different chiplevel delays. The LS codes employed in our system have both zero out-of-phase Auto-Correlations (ACLs) and zero Cross-Correlations (CCLs) within their Interference Free Window (IFW). Without loss of generality, we assume that the signal received from the desired user is perfectly synchronized, but all the other signals have random delays with respect to the desired user's signal within the range  $[-\tau_{max}, \tau_{max}]$ . The delayed signal received from the k-th user is represented by  $\bar{\mathbf{y}}_k(i)$ . Then the signal received at the *m*-th RN may be modelled as:

$$\mathbf{r}^{m}(i) = \sum_{k=1}^{K} \bar{\mathbf{y}}_{k}(i) \cdot h_{k}^{SR_{m}}(i) + \mathbf{n}_{SR_{m}}(i), \qquad (6)$$

where the fading  $h_k^{SR_m}(i)$  obeys the Rayleigh distribution, and the chip-level Additive White Gaussian Noise (AWGN) vector  $\mathbf{n}_{SR_m}(i)$  has  $(T \cdot L_S \times 1)$  elements, which has a zero mean and a variance of  $N_0^{R_m}$  in each dimension. The composite K-user received signal vector also has  $(T \cdot L_S \times 1)$  elements.

The first processing step of the RNs is to de-spread the desired user's signal. Assuming that the m-th RN is dedicated to the k-th user, then the de-spreading operation may be expressed as:

$$\mathbf{d}_{k}^{SR_{m}}(i) = \frac{1}{\sqrt{G_{S}}} (\mathbf{I}_{\mathbf{T}} \otimes \bar{\mathbf{c}}_{k})^{T} \cdot \mathbf{r}^{m}(i) = \sqrt{P_{S}} \mathbf{s}_{k}(i) \cdot h_{k}^{SR_{m}}(i) + \mathbf{J}_{k}^{SR_{m}}(i) + \mathbf{N}_{k}^{SR_{m}}(i),$$
(7)

where the vector  $\mathbf{d}_{k}^{SR_{m}}(i) = [d_{k}^{SR_{m1}}(i), \cdots, d_{k}^{SR_{mT}}(i)]^{T}$  has a size of  $(T \times 1)$ . The MUI term  $\mathbf{J}_{k}^{SR_{m}}(i)$  of Eq. (7) is supposed to be zero, when the maximum user-signal delay  $\tau_{max}$  is smaller

than the IFW length of the employed LS code. Otherwise, the MUI imposed on the RNs would propagate to the DN, hence resulting in a severely degraded overall performance. The noise term  $\mathbf{N}_{k}^{SR_{m}}(i) = \frac{1}{\sqrt{G_{S}}} (\mathbf{I_{T}} \otimes \bar{\mathbf{c}}_{k})^{T} \cdot \mathbf{n}_{SR_{m}}(i)$  has a zero mean and a variance of  $N_{0}^{R_{m}}$ . After de-spreading, the RNs perform re-spreading based on DSTS. The *M* RNs act as the DSTS VAA, in which *T* symbols are spread using *T*  $L_{R}$ -chip spreading codes at the *M* RNs according to the orthogonal  $\tilde{G}_{M}$  structure of Eq. (4). In case of M = 2, the DSTS mapping at the two RNs may be expressed as:

$$\mathbf{y}_{k}^{R_{1}}(i) = \frac{\alpha_{1}}{\sqrt{G_{R}}} \left[ d_{k}^{SR_{11}}(i) \cdot \widetilde{\mathbf{c}}_{k}^{1} + d_{k}^{SR_{12}}(i) \cdot \widetilde{\mathbf{c}}_{k}^{2} \right],$$
  
$$\mathbf{y}_{k}^{R_{2}}(i) = \frac{\alpha_{2}}{\sqrt{G_{R}}} \left[ d_{k}^{SR_{22}}(i) \cdot \widetilde{\mathbf{c}}_{k}^{1} - d_{k}^{SR_{21}}(i) \cdot \widetilde{\mathbf{c}}_{k}^{2} \right],$$
(8)

where the T spreading codes used at the RNs  $\{\{\mathbf{\tilde{c}}_{k}^{t}\}_{t=1}^{T}\}_{k=1}^{K}$ , which have  $L_{R}$  chips and a power of  $G_{R}$ , are different for each of the K users. Hence  $(T \times K)$  spreading codes<sup>2</sup> are required for the RNs. The transmission matrix  $\mathbf{y}_{k}^{R_{m}}(i)$  at the m-th RN has  $(L_{R} \times 1)$  elements, which implies that each transmission block requires  $L_{R}$  chip periods and a single transmit antenna. The amplification factor of the m-th AF relay is given by:

$$\alpha_m = \sqrt{\frac{P_R}{M \cdot (P_S \cdot \sigma_{SR_m}^2 + N_0^{R_m})}},\tag{9}$$

where  $\sigma_{SR_m}^2$  represents the power of the *m*-th SR (SR<sub>m</sub>) link.

The DSTS employing four RNs may be conducted in the same way, as documented in [10]. For the case of M = 3, the 4-th RN employed in [10] is not activated.

## C. Destination Node

1) <u>Broadcast Interval</u>: At the DN, the CDMA-chips received from the SNs during the broadcast interval may be expressed as:

$$\mathbf{r}^{SD}(i) = \sum_{k=1}^{K} \bar{\mathbf{y}}_k(i) \cdot \mathbf{H}_k^{SD}(i) + \mathbf{n}_{SD}(i), \qquad (10)$$

where  $\{\bar{\mathbf{y}}_k(i)\}_{k=1}^K$  denotes the asynchronous signals received from the *K* users. The non-dispersive channel vector  $\mathbf{H}_k^{SD}(i)$  is of size  $(1 \times N)$ . The received signal vector  $\mathbf{r}^{SD}(i)$  has  $(TL_S \times N)$  elements and the AWGN matrix  $\mathbf{n}_{SD}(i)$  has the same size, as well as a zero mean and a variance of  $N_0^D$  in each dimension.

The de-spreading operation carried out at the DN is the same as the one at the RNs, which is given by:

$$\mathbf{d}_{k}^{SD}(i) = \frac{1}{\sqrt{G_{S}}} (\mathbf{I}_{\mathbf{T}} \otimes \bar{\mathbf{c}}_{k})^{T} \cdot \mathbf{r}^{SD}(i) = \sqrt{P_{S}} \mathbf{s}_{k}(i) \cdot \mathbf{H}_{k}^{SD}(i) + \mathbf{J}_{k}^{SD}(i) + \mathbf{N}_{k}^{SD}(i),$$
(11)

where only the desired user is assumed to be synchronized. The detected signal matrix  $\mathbf{d}_{k}^{SD}(i)$  has  $(T \times N)$  elements, and the despread AWGN term  $\mathbf{N}_{k}^{SD}(i) = (\mathbf{I}_{\mathbf{T}} \otimes \mathbf{\bar{c}}_{k})^{T} \cdot \mathbf{n}_{SD}(i)$  has the same size as well as a variance of  $N \cdot N_{0}^{D} \cdot \mathbf{I}_{\mathbf{T}}$ .

2) <u>Cooperation Interval</u>: The DSTS-chips received from the RNs during the cooperation interval are given by:

$$\mathbf{r}^{RD}(i) = \sum_{k=1}^{K} \sum_{m=1}^{M} \bar{\mathbf{y}}_{k}^{Rm}(i) \cdot \mathbf{H}_{k}^{RD_{m}}(i) + \mathbf{n}_{RD}(i), \quad (12)$$

where  $\{\{\bar{\mathbf{y}}_{k}^{R_{m}}(i)\}_{m=1}^{M}\}_{k=1}^{K}$  denotes the randomly delayed signal received from all  $N_{R} = MK$  RNs. The non-dispersive channel

vector  $\mathbf{H}_{k}^{RD_{m}}(i)$  is of size  $(1 \times N)$ . The received signal matrix  $\mathbf{r}^{RD}(\mathbf{i})$  has  $(L_{R} \times N)$  elements, while the AWGN matrix  $\mathbf{n}_{RD}(i)$  has the same size and a variance of  $N_{0}^{D}$  in each dimension.

The DN then de-spreads the received signal using T spreading codes for the desired user. The resultant signal is represented by  $\{\mathbf{d}_{k}^{RD_{t}}\}_{t=1}^{T}$ . The de-spreading operation may be expressed as:

$$\mathbf{d}_{k}^{RD_{t}}(i) = \frac{1}{\sqrt{G_{R}}} (\tilde{\mathbf{c}}_{k}^{t})^{T} \cdot \mathbf{r}^{RD}(i), \qquad (13)$$

where  $\mathbf{d}_{k}^{RD_{t}}(i)$  has a size of  $(1 \times N)$ . For the case of using M = 2 RNs, the de-spread signals may be formulated as:

$$\begin{aligned} \mathbf{d}_{k}^{RD_{1}}(i) &= \frac{1}{\sqrt{G_{R}}} (\widetilde{\mathbf{c}}_{k}^{1})^{T} \cdot \mathbf{r}^{RD}(i) \\ &= \alpha_{1} \sqrt{P_{S}} \cdot s_{k}^{1}(i) \cdot h_{k}^{SR_{1}}(i) \cdot \mathbf{H}_{k}^{RD_{1}}(i) \\ &+ \alpha_{2} \sqrt{P_{S}} \cdot s_{k}^{2}(i) \cdot h_{k}^{SR_{2}}(i) \cdot \mathbf{H}_{k}^{RD_{2}}(i) + \mathbf{J}_{k}^{RD_{1}}(i) + \mathbf{N}_{k}^{RD_{1}}(i), \\ \mathbf{d}_{k}^{RD_{2}}(i) &= \frac{1}{\sqrt{G_{R}}} (\widetilde{\mathbf{c}}_{k}^{2})^{T} \cdot \mathbf{r}^{RD}(i) \\ &= \alpha_{1} \sqrt{P_{S}} \cdot s_{k}^{2}(i) \cdot h_{k}^{SR_{1}}(i) \cdot \mathbf{H}_{k}^{RD_{1}}(i) \\ &- \alpha_{2} \sqrt{P_{S}} \cdot s_{k}^{1}(i) \cdot h_{k}^{SR_{2}}(i) \cdot \mathbf{H}_{k}^{RD_{2}}(i) + \mathbf{J}_{k}^{RD_{2}}(i) + \mathbf{N}_{k}^{RD_{2}}(i), \end{aligned}$$

where the noise term  $\{\mathbf{N}_{k}^{RD_{t}}(i)\}_{t=1}^{T}$  has a variance of  $N \cdot (N_{0}^{D} + \sum_{m=1}^{M} \alpha_{m}^{2} \sigma_{RD_{m}}^{2} N_{0}^{Rm})$ , where  $\sigma_{RD_{m}}^{2}$  represents the power of the *m*-th RD (RD<sub>m</sub>) link.

The MUI terms at the destination, i.e.  $\mathbf{J}_{k}^{SD}(i)$  of Eq. (12) and  $\{\mathbf{J}_{k}^{RD_{t}}(i)\}_{t=1}^{T}$  of Eqs. (13)-(14), are supposed to be zero, when  $\tau_{max}$  is smaller than the IFW length of the LS code employed, and these terms will be ignored by the detector.

3) <u>MSDD/MSDSD</u>: Due to lack of space, we only portray the MSDD/MSDSD case in this paper. A summary of CDD may be found in [9].

Similar to the DSTBC received signal model, Eq. (11) may be formulated as:

$$\mathbf{Y}_{k}^{SD}(i) = \widetilde{G}_{M}(\mathbf{s}_{k}(i)) \cdot \widetilde{\mathbf{H}}_{k}^{SD}(i) + \mathbf{N}_{k}^{SD}(i), \qquad (15)$$

where  $\mathbf{Y}_{k}^{SD}(i) = \mathbf{d}_{k}^{SD}(i)$  has  $(T \times N)$  elements. The equivalent SD links' fading channel matrix  $\widetilde{\mathbf{H}}_{k}^{SD}(i) = \left[\left(\sqrt{P_{S}}\mathbf{H}_{k}^{SD}(i)\right)^{T}, \left(\mathbf{0}_{(\mathbf{M}-1)\times\mathbf{N}}\right)^{T}\right]^{T}$  represents an equivalent  $(M \times N)$ -element channel, where the fading coefficients between the (M-1) hypothetical transmit antennas and the N receive antennas are all zero.

Similarly, a universal model of Eq. (13) is given by:

$$\mathbf{Y}_{k}^{RD}(i) = \widetilde{G}_{M}(\mathbf{s}_{k}(i)) \cdot \widetilde{\mathbf{H}}_{k}^{RD}(i) + \mathbf{N}_{k}^{RD}(i), \qquad (16)$$

where  $\mathbf{Y}_{k}^{RD}(i) = \left[ \left( \mathbf{d}_{k}^{RD_{1}}(i) \right)^{T}, \cdots, \left( \mathbf{d}_{k}^{RD_{T}}(i) \right)^{T} \right]^{T}$  has  $(T \times N)$  elements, while the equivalent AWGN term  $\mathbf{N}_{k}^{RD}(i) = \left[ \left( \mathbf{N}_{k}^{RD_{1}}(i) \right)^{T}, \cdots, \left( \mathbf{N}_{k}^{RD_{T}}(i) \right)^{T} \right]^{T}$  has the same size. The equivalent fading channel matrix  $\widetilde{\mathbf{H}}_{k}^{RD}(i) = \left[ \left( \alpha_{1} \sqrt{P_{S}} h_{k}^{SR_{1}}(i) \mathbf{H}_{k}^{RD_{1}}(i) \right)^{T}, \cdots, \left( \alpha_{M} \sqrt{P_{S}} h_{k}^{SR_{M}}(i) \mathbf{H}_{k}^{RD_{M}}(i) \right)^{T} \right]^{T}$  has  $(M \times N)$  elements.

Upon combining Eq. (15) and Eq. (16), the overall universal model may be formulated as:

$$\mathbf{Y}_{k}(i) = \left[\mathbf{I}_{2} \otimes \widetilde{G}_{M}(\mathbf{s}_{k}(i))\right] \cdot \mathbf{H}_{k}(i) + \mathbf{N}_{k}(i), \qquad (17)$$

where the combined spread signal matrix  $\mathbf{Y}_{k}(i) = \left[\left(\mathbf{Y}_{k}^{SD}(i)\right)^{T}, \left(\mathbf{Y}_{k}^{RD}(i)\right)^{T}\right]^{T}$  has  $(2T \times N)$  elements, and

<sup>&</sup>lt;sup>2</sup>We note that in case of LS codes having an IFW the system may become 'code-limited', rather than interference-limited, since the number of LS codes is more limited than that of say Walsh-Hadamard codes.

 $|\mathbf{I_2} \otimes \widetilde{G}_M(\mathbf{s}_k(i))|$  forms an equivalent cooperative transmission matrix. The equivalent cooperative fading channel matrix  $\mathbf{H}_{k}(i) = \left[ \left( \widetilde{\mathbf{H}}_{k}^{SD}(i) \right)^{T}, \left( \widetilde{\mathbf{H}}_{k}^{RD}(i) \right)^{T} \right]^{T} \text{ having a size of } (2M \times N)$  incorporates all the cooperative links. The equivalent AWGN matrix  $\mathbf{N}_{k}(i) = \left[ \left( \mathbf{N}_{k}^{SD}(i) \right)^{T}, \left( \mathbf{N}_{k}^{RD}(i) \right)^{T} \right]^{T}$  of  $(2T \times N)$  elements contains the AWGN both at the RNs and at the DN.

Instead of making a decision on a single information matrix based on two consecutive received signal matrices as in CDD, our MSDD/MSDSD scheme observes  $N_w$  received blocks and makes a joint decision based on  $(N_w - 1)$  information matrices. Therefore, the equivalent de-spread signal model of our MSDD/MSDSD scheme may be expressed as:

$$\mathbf{Y}_k = \hat{\mathbf{S}}_k \cdot \mathbf{H}_k + \mathbf{N}_k, \tag{18}$$

where the  $N_w$  blocks based de-spread signal matrix  $\mathbf{Y}_k = \begin{bmatrix} (\mathbf{Y}_k(i))^T, \cdots, (\mathbf{Y}_k(i+N_w-1))^T \end{bmatrix}^T = \begin{bmatrix} (\mathbf{Y}_k^1)^T, \cdots, (\mathbf{Y}_k^{N_w})^T \end{bmatrix}^T$ has  $(2TN_w \times N)$  elements. The accumulated fading channel matrix  $\mathbf{H}_k$  and the accumulated AWGN matrix  $\mathbf{N}_k$  are modelled in the same way, and their sizes are  $(2MN_w \times N)$  and  $(2TN_w \times N)$ , respectively. The  $N_w$ -block transmit signal matrix  $\hat{\mathbf{S}}_k = diag \left\{ \mathbf{I}_2 \otimes \widetilde{G}_M(\mathbf{s}_k(i)), \cdots, \mathbf{I}_2 \otimes \widetilde{G}_M(\mathbf{s}_k(i+N_w-1)) \right\} =$  $diag\left\{ \hat{\mathbf{S}}_{k}^{1},\cdots,\hat{\mathbf{S}}_{k}^{N_{w}}
ight\}$  has  $(2TN_{w} imes2MN_{w})$  elements.

MSDD/MSDSD aims for maximizing the a posteriori probability in terms of [12], [13]:

$$\Pr\left(\mathbf{Y}_{k} \mid \hat{\mathbf{S}}_{k}\right) \propto \exp\left(-Tr\left\{\mathbf{Y}_{k}^{H}(\mathbf{R}_{\mathbf{Y}\mathbf{Y}})^{-1}\mathbf{Y}_{k}\right\}\right),$$
(19)

where the autocorrelation matrix of  $Y_k$  of Eq. (18) is given by:

$$\mathbf{R}_{YY} = \hat{\mathbf{S}}_k \mathbf{R}_{HH} \hat{\mathbf{S}}_k^H + \mathbf{R}_{NN}$$
  
=  $\hat{\mathbf{S}}_k \mathbf{C} \hat{\mathbf{S}}_k^H$ , (20)

and the channel correlation matrix  $\mathbf{C} = \mathbf{R}_{HH} + \frac{1}{M}\mathbf{R}_{NN}$  has  $(2MN_w \times 2MN_w)$  elements, because  $\hat{\mathbf{S}}_k$  in Eq. (19) is a scaled unitary matrix satisfying  $\hat{\mathbf{S}}_k \hat{\mathbf{S}}_k^H = M \cdot \mathbf{I}_{2\mathbf{T} \times \mathbf{N}_w}$ , which therefore does not affect the correlations in  $\mathbf{R}_{NN}$ .

According to Clarke's model, the correlation factors of the flat Rayleigh fading are determined by  $\varphi(i) = \varepsilon \{h(t) \cdot h^*(t+i)\} =$  $J_0(2\pi i f_d)$ , where  $J_0$  and  $f_d$  denote the zero-order Bessel function of the first kind and the normalized Doppler frequency, respectively. Therefore, the flat fading channel autocorrelation matrix in Eq (20) is given by  $\mathbf{R}_{HH} = toeplitz \{ \Gamma_0, \Gamma_1, \cdots, \Gamma_{N_w-1} \},$  where 
$$\begin{split} &\Gamma_{i} = diag \{ \Gamma_{i}^{SD}, \Gamma_{i}^{RD} \}, \text{ and both } \Gamma_{i}^{SD} = NP_{S}\sigma_{SD}^{2}\varphi_{SD}(i)\mathbf{I_{T}} \text{ and} \\ &\Gamma_{i}^{RD} = \frac{1}{M} \left[ \sum_{m=1}^{M} N\alpha_{m}^{2}P_{S}\sigma_{SR_{m}}^{2}\sigma_{RD_{m}}^{2}\varphi_{SR_{m}}(i)\varphi_{RD_{m}}(i) \right] \cdot \mathbf{I_{T}} \\ &\text{have} \quad (T \times T) \quad \text{elements} \quad \text{Furthermore, the } \quad \text{AWGN} \end{split}$$
autocorrelation matrix is given by  $\mathbf{R}_{NN} = \mathbf{I}_{\mathbf{N}_{\mathbf{w}}} \otimes \left[ diag \left\{ N \cdot N_0^D, N \cdot (N_0^D + \sum_{m=1}^M \alpha_m^2 \sigma_{RD_m}^2 N_0^{R_m}) \right\} \otimes \mathbf{I}_{\mathbf{M}} \right].$ According to Eqs. (19)-(20), the ML decision metric of the MSDD

may be formulated as:

$$\hat{\mathbf{S}}_{k}^{ML} = \arg\min_{\hat{\mathbf{S}}_{k}} \| \mathbf{F} \hat{\mathbf{S}}_{k}^{H} \mathbf{Y}_{k} \|^{2}, \qquad (21)$$

where the upper triangular matrix  $\mathbf{F}$  is generated from the decomposition of  $\mathbf{C}^{-1} = \mathbf{F}^{H} \mathbf{F}$ , which may be defined as:

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_{1,1} & \mathbf{F}_{1,2} & \dots & \mathbf{F}_{1,N_w} \\ \mathbf{0}_{2\mathbf{T} \times 2\mathbf{T}} & \mathbf{F}_{2,2} & \dots & \mathbf{F}_{2,N_w} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{2\mathbf{T} \times 2\mathbf{T}} & \mathbf{0}_{2\mathbf{T} \times 2\mathbf{T}} & \dots & \mathbf{F}_{N_w,N_w} \end{bmatrix}, \quad (22)$$

where the component matrix  $\{\{\mathbf{F}_{i,j}\}_{i=1}^{N_w}\}_{j=i}^{N_w}$  has  $(2M \times 2M)$  elements. Thus, the decision metric in Eq. (21) may be decomposed

$$\| \mathbf{F} \hat{\mathbf{S}}_{k}^{H} \mathbf{Y}_{k} \|^{2} = \sum_{i=1}^{N_{w}} \| \sum_{j=i}^{N_{w}} \mathbf{F}_{i,j} \left( \hat{\mathbf{S}}_{k}^{j} \right)^{H} \mathbf{Y}_{k}^{j} \|^{2} .$$
(23)

Then the MSDSD algorithm may be formulated as:

$$d_i^2 = \sum_{\hat{i}=i}^{N_w} \|\sum_{j=\hat{i}}^{N_w} \mathbf{F}_{\hat{i},j} \left( \hat{\mathbf{S}}_k^j \right)^H \mathbf{Y}_k^j \|^2$$
  
=  $d_{i+1}^2 + \|\mathbf{\Delta}_i + \mathbf{K}_i\|^2$ , (24)

where the Partial Euclidean Distance (PED)  $\{d_i^2\}_{i=1}^{N_w - 1}$  lies within the decoding sphere. Throughout the Sphere Decoding (SD) procedure used [14], [15],  $\mathbf{\Delta}_i = \mathbf{F}_{i,i} \left(\hat{\mathbf{S}}_k^i\right)^H \mathbf{Y}_k^i$  should be updated, and  $\mathbf{K}_{i} = \sum_{j=i+1}^{N_{w}} \mathbf{F}_{i,j} \left( \hat{\mathbf{S}}_{k}^{j} \right)^{H} \mathbf{Y}_{k}^{j}$  should be accumulated. The detailed SD search strategy may be found in [14], [15].

## **III. PERFORMANCE RESULTS AND DISCUSSION**

In this section, we present our performance results for the proposed MSDSD aided CDSTS scheme designed for the asynchronous CDMA uplink. BPSK signalling is employed. All the channel powers are normalized to unity in order to provide a fair comparison between the corresponding non-cooperative and cooperative schemes. Furthermore, the power allocated to the SN and to the RNs was set to  $P_S = P_R = \frac{1}{2}$ , as suggested in [19]. It was also suggested in [19] that the attainable performance becomes better, if the RNs are close to the DN in AF cooperative schemes. Thus, we assume that the noise power imposed at the RNs and at the DN are the same.

Without loss of generality, M = 2 RNs are employed for each of the K = 4 users, hence the total number of RNs becomes  $N_R = 8$ . As summarized in [11], a LS code is represented by  $LS(N_{LS}, P_{LS}, W_0)$ , which implies that the code is constructed from an orthogonal complementary code set having a codeword length of  $N_{LS}$  with the aid of a  $(P_{LS} \times P_{LS})$ -dimensional Walsh-Hadamard matrix, and as a result,  $P_{LS}$  users could be supported. The corresponding IFW length is given by  $\min\{N_{LS} - 1, W_0\}$ , and the code length is given by  $L = N_{LS}P_{LS} + 2W_0$  [10]. The LS(8,4,7) code is employed for the K = 4 SNs, and the LS(8,8,7) code is used for the  $N_R = 8$  RNs.

The BER performance of CDD operating in slow fading channels is shown in Fig. 2. It can be seen that the proposed scheme achieves the expected diversity gain and outperforms the corresponding noncooperative scheme, when the maximum delay obeys  $\tau_{max} < 7T_c$ . However, the attainable performance degrades severely, when  $\tau_{max}$ exceeds the IFW length, and the performance of the cooperative scheme becomes even worse than that of the non-cooperative scheme, when Gold codes are employed. This is because the non-zero CCLs of Gold codes, or those of the LS code, which fell outside the IFW, induce non-zero MUI in Eqs. (7), (11) and (13).

Fig. 3 compares our cooperative DSTS scheme and the co-located DSTS scheme of [3]. In the absence of antenna correlation, the colocated DSTS scheme outperforms cooperative DSTS. This trend prevails even if our cooperative scheme has a higher diversity order, because the noise imposed at the RNs is amplified and forwarded to the DN. However, it is also demonstrated by Fig. 3 that the colocated DSTS gradually loses its beneficial diversity gain in realistic spatially correlated fading channels, and the proposed cooperative scheme becomes more advantageous, as a benefit of its distributed design.



Fig. 2. BER performance of the CDD aided CDSTS in the asynchronous CDMA uplink, using different spreading codes, for  $f_d = 0.001$ .



Fig. 3. BER performance of the CDD aided CDSTS in the asynchronous CDMA uplink, compared to the co-located DSTS scheme of [3].

When the fading channel fluctuates rapidly, an irreducible error floor is formed for the CDD, which is evidenced by Fig. 4. As a remedy, MSDSD is introduced. Fig. 4 demonstrates that the performance of the non-cooperative scheme remains worse than that of the cooperative scheme, even when we use MSDSD in conjunction with as wide a detection window as  $N_w = 11$  for the non-cooperative scheme, while employing low-complexity CDD for the cooperative scheme. Finally, the MSDSD scheme successfully mitigates the pronounced error floor encountered in rapidly fading channels.

## **IV. CONCLUSIONS**

In this contribution, we proposed a MSDSD aided CDSTS scheme for the asynchronous CDMA uplink, where neither CSI nor symbollevel synchronization is required. We demonstrated that the proposed scheme outperforms its non-cooperative counterparts by using multiple relays based on DSTS. Furthermore, the irreducible error floor of CDD encountered in rapidly fading channels is successfully mitigated by our proposed MSDSD.

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Fig. 4. BER performance of the MSDSD aided CDSTS in the asynchronous CDMA uplink, for  $f_d = 0.03$ .

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