Review and Simulation of Quantum Teleportation

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Abstract
Data communication technology has been actively researched for many years. As the conjunction of computer science and physics, quantum information brings brand new fundamentals with the term quantum communication. In this report we introduce a protocol in quantum communication called quantum teleportation, with explanations about its motivation and possible applications. Previous and current research achievements are summarised and evaluated in the authors' perspective. After that a simulation program of quantum teleportation was implemented, tested and evaluated, giving constructive conclusions in the end.

Keywords: quantum information, quantum teleportation, quantum circuit, entanglement, review, software simulation

1. Introduction

The subject of quantum information theory (QIT) nowadays is a fast developing area with increasing investigations and researches around the globe. In this report we will focus on a subarea in QIT named quantum teleportation (QT) [1–5]. The objective of QT is to transmit an unknown quantum state over a distance, and it is one of the few quantum communication protocols invented so far .

One of the most important elements in transition from classical information theory to QIT is to understand the concept of quantum bit, or qubit [6]. A qubit represents the state of a quantum system with two orthogonal basis: $|0\rangle$ and $|1\rangle$, where the special symbols are called bra-ket notation [7]. An interesting property of qubit is its probabilistic superposition of two states, which says it can hold $|0\rangle$ and $|1\rangle$ at the same time with certain probabilities to fall into one of the state when is measured. This means, we never know a qubit's true state before measuring it, while any measurement behaviours will break the superposition state into a pure state (distinctive $|0\rangle$ or $|1\rangle$). The notation of a qubit is written as:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \qquad (1)$$

where α and β are called qubit amplitudes and $|\alpha|^2 + |\beta|^2 = 1$.

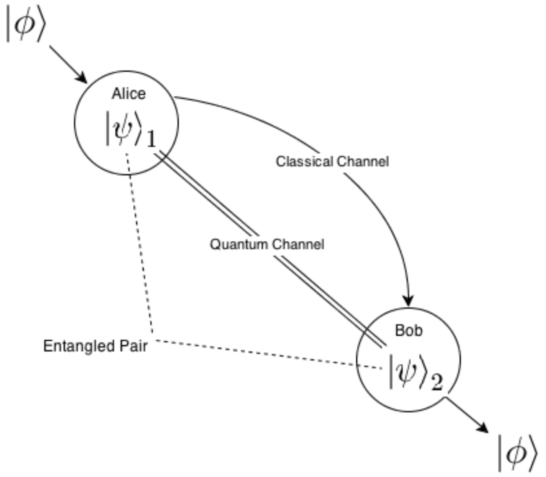


Fig. 1 Quantum teleportation

The theoretical report of quantum teleportation was first published in 1993 by six scientists [5]. They proved the teleportation of a qubit $|\phi\rangle$ from "Alice" to "Bob" is possible by destroying the qubit which Alice at hand then restore it at Bob's side, without obtaining any information about the quantum state $|\phi\rangle$. The process can be explained as follows. Say Alice has three qubits in photon system, in which two of them $|\psi\rangle_1$ and $|\psi\rangle_2$ are entangled in the Bell state [8] in polarisation:

$$|\Psi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2) \qquad (2)$$

and another one $|\phi\rangle$ is in an unknown state which she will send its information to Bob:

$$|\phi\rangle = a|\uparrow\rangle + b|\downarrow\rangle. \qquad (3)$$

The arrows stands for the polarisation direction of the quantum state, where an upward arrow means 90° and a downward one means -90°. Firstly Alice sends one of the entangled qubit to Bob via some quantum channel, then entangle the remaining two qubits by performing a joint Bell-state measurement [9]. The complete state of the three qubits is now:

$$\begin{aligned}|\Psi\rangle_{123} =& \frac{a}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2|\uparrow\rangle_3 - |\downarrow\rangle_1|\uparrow\rangle_2|\uparrow\rangle_3)\\ &+ \frac{b}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2|\downarrow\rangle_3 - |\downarrow\rangle_1|\uparrow\rangle_2|\downarrow\rangle_3).\end{aligned} \tag{4}$$

Subscript 3 stands for the qubit of the unknown state. According to the Bell state basis we can then rewrite the equation into:

$$\begin{aligned}|\Psi\rangle_{123} = \frac{1}{2}[&|\Psi^-\rangle_{13}(-a|\uparrow\rangle_2 - b|\downarrow\rangle_2)\\ &+|\Psi^+\rangle_{13}(-a|\uparrow\rangle_2 + b|\downarrow\rangle_2)\\ &+|\Phi^-\rangle_{13}(a|\downarrow\rangle_2 + b|\uparrow\rangle_2)\\ &+|\Phi^+\rangle_{13}(a|\downarrow\rangle_2 - b|\uparrow\rangle_2)].\end{aligned} \tag{5}$$

This explains that, there should be four possible measurement results with equal probability of 1/4. At this step the qubit $|\psi\rangle_2$, which Bob has, is supposed to be projected into one of the four pure states shown in Eq.(5). It is not hard to see that $|\psi\rangle_2$ now has very similar state to the original $|\phi\rangle$, where one of the cases is exactly the same. In other words, the teleportation is already successful with 25% chance. With the other three cases we can perform unitary transformations to $|\psi\rangle_2$ as long as the measurement outcome at Alice's side is provided, which finally produces an exact replica of the destroyed $|\phi\rangle$. The process of sending Alice's measurement information is called active feed-forward, and is usually achieved with a classical data channel. The whole process of quantum teleportation is also illustrated in Fig. 1.

Following that discovery other scientists realised the idea and demonstrated QT experiments in various quantum systems such as photons, nuclear spins and trapped ions [10, 11]. Photons is the most widely used system since it is found to have little decoherence from noisy environment and are easy to manipulate as well as transmit. The early methods of distributing photons were mainly through optical fibres. Although experiments with fibres were successful, even with cutting-edge techniques [12], optical fibres were still left behind due to their high photon loss and decoherence factor. Instead free-space optical link was proved to be the successor, mainly because of its low negative effects from atmosphere, which brings expressively longer transmission distance (144 kilometres maximum so far) [2, 3, 13].

2. Motivation and Possible Applications

Quantum computing is the foreseeable future of computer science, it utilises quantum mechanical phenomena and relies on quantum information theory. The direct communication in qubits between different individuals is an essential part in distributed quantum computing, which also introduces the need for a global quantum network. However, the sending of quantum states in classical means is impossible, which can be explained in the following arguments [14]:

- Physical transport of qubits.
  Fragile quantum states would be easily corrupted by very little interruptions during the way, which is not recommended.

- Broadcast of quantum information.
  No-broadcast theorem, a corollary of the no-cloning theorem, states that it is impossible to create a state that both of its parts are the same as the original state.

- Measure the quantum going to be sent, then send its information in classical manner and creates a new quantum on the receiver side according to the information.
  Since we can only perform a single measurement of any quantum before disturbing it, and due to Heisenberg's uncertainty principle, we cannot measure the complete states of a quantum. This is also explained in the no-teleportation theorem [15].

Quantum teleportation is the first and only solution so far, it gives the feasibility to transfer unknown quantum states over long distances. The first realistic application of this technology is in quantum cryptography.

Nowadays the most popular and reliable method for data encryption is to separate a private encryption key depending on the data, and transfer the key individually. Quantum cryptography [16] is a rather old field compared to QT, its main idea is to transmit the private key via quantum states, where majority experiments uses polarisation of photons. The most famous protocol, BB84 [16], is a method that calculates private key on both sender and receiver sides, depending on the photon polarisations measured. It is said to be immune to eavesdropping because interception and retransmission by the eavesdropper cannot extract useful data. However, in that case, the transmission will be disturbed and the noise ratio greatly increases. Quantum teleportation solves this problem by completely forbidding eavesdropping and is called "the ultimate solution to quantum cryptography" [16].

Apart from quantum key distribution (QKD), the field of quantum secure direct communication (QSDC) is being actively researched during recent years [17–21], where many of them use teleportation. The QSDC protocol invented by F.G. Deng *et al.* in 2008 [17] summarised previous techniques and produced a novel method that uses superdense coding. The protocol inherited some merits from that in T. Gao *et al.*'s teleportation protocol [19], and could be adopted with QT as well. They claimed in their method that any eavesdropping behaviours will be twice easier to detect compared to previous BB84 QKD protocol, for the reason that those behaviours will double the error rate. Details of this protocol will be introduced in section 4.

Another discovery in quantum teleportation is entanglement swapping. This technique achieves the entanglement of qubits that never interacted. Consider the following case as shown in Fig. 2, we have four qubits, $|\phi\rangle_1$, $|\phi\rangle_2$, $|\psi\rangle_1$ and $|\psi\rangle_2$ distributed as the graph illustrates. $|\phi\rangle_1$ and $|\phi\rangle_2$ are entangled and so are $|\psi\rangle_1$ and $|\psi\rangle_2$, where $|\phi\rangle_2$ and $|\psi\rangle_1$ are in the same place. The idea is, in short, to teleport the state of $|\phi\rangle_2$ to $|\psi\rangle_2$ by performing projective measurement (Bell-state measurement [9]) on $|\phi\rangle_2$ and $|\psi\rangle_1$. As a result, $|\phi\rangle_1$ and $|\psi\rangle_2$ are now entangled [22]. This technique was applied in quantum repeater [23] and explored to fit with a secure direct communication protocol [18].

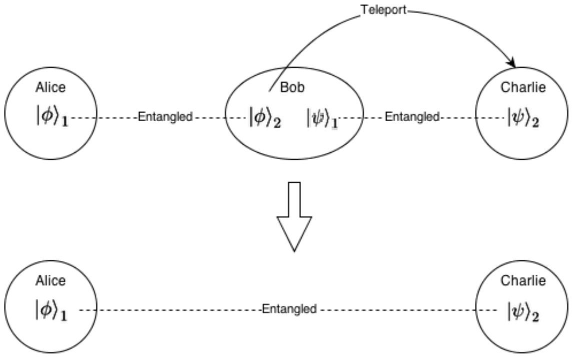


Fig. 2 Entanglement swapping

In the research of quantum network, QT can be used as a communication protocol at the cost of an EPR pair and two classical bits per qubit [1]. This requires entanglement distribution by routers between Alice and Bob who do not share direct connections, which means it is possible to establish virtual connections between nodes without a direct quantum channel between them [1].

3. Challenges

A major challenge in quantum teleportation is the significant short coherence time between entangled pairs. A qubit is said to be in coherence when it is in superposition, that is, simultaneously holds values 1 and 0. Any perturbation from the environment will result the qubit collapse into a pure state and terminate the entanglement [24]. At the current level of research, we have to use entanglement distribution via optical channels, which is a delicate and challenging task in QT, it is also one of the main reasons that make QT not a feasible solution to the current field of quantum communication [3]. This problem will hopefully be solved with the forthcoming technology of quantum memory. In this case, entangled quanta can be stored in separate quantum memories and be kept at different locations for significantly longer time, which brings future QT further advantages:

- QT is available even when quantum channel has low quality, or
- When the location of Bob is unknown, means a broadcast of classical information from Alice.

The current best achievement in practical quantum memory is of 1.75s decoherence time with 90% fidelity, which is still far from what QT requires [25]. The authors developed a coherence transfer scheme that involves a processing qubit (electron spin) and a memory qubit (nuclear spin). In the writing process, a coherent electron spin state is generated then stored into a nuclear spin, and vice versa for the reading process.

Experimental free-space long-distance QT face many difficulties, especially in the manipulation of quantum channel. For instance, one is the extremely low signal-to-noise ratio. To cope with this, a successful experiment in 2012 utilised: frequency-uncorrelated polarization-entangled photon pair source, ultra-low-noise large-active-area single-photon detectors and entanglement-assisted clock synchronisation, while the experiments could only be carried out during night [3].

4. History and Current Development

Starting with the first discovery of quantum teleportation by Bennett *et al* [5], significant amount of research has been done in the relevant areas in all over the world. It is worth mention that theoretical discoveries are far beyond what have been achieved experimentally.

The first experimental QT [11] happened in 1997, they used photon pairs entangled in polarisation. At that time the techniques for any kind of entanglement were immature, for which they used a method called type-II parametric down-conversion that produces entangled photons in orthogonal polarisations [26]. However in that experiment they did not use classical information at the Bob's side, that means only one case in the four possible Bell-state measurement outcomes is considered (the case where Bob need to apply an **I** gate).

In the same year a theoretical report [27] explained the process to teleport continuous spectra ("manifestly quantum states of electromagnetic field"), with two ancillary highly squeezed two-mode electromagnetic fields as the entangled states, which was a valuable step in quantum teleportation from discrete-variable systems to continuous-variable systems. They also managed to obtain accurate fidelity of entanglement in an infinite dimensional Hilbert space with absolute efficiency, compared to parametric down-conversion which only has rare success rate.

In the following year a method of QT with a Greenberger-Horne-Zeilinger state triplet [28] was published [29]. In that experiment three distant entities were involved, and one can teleport an unknown state to either of other two places. That technique was further expanded in 2004 [30]. The authors formulated a technique to teleport strings of qubits from one location to another via a many-agent network. A notable improvement was that, regardless of the amount of qubits to be teleported, every agent in the network only need one GHZ state qubit, perform one Hadamard gate transform and send one classical bit to the receiver.

The multi-qubit teleportation scheme was also explored in the context of secure transmission in that year. A quantum secure direct communication (QSDC) protocol using controlled quantum teleportation was discovered [19]. They used a triplet of entangled photons which was different from GHZ, and invented a new triplet-based controlled quantum teleportation protocol, with the third entity Charlie as a supervisor of the transmission between Alice and Bob. The new protocol can be explained as following.

Again we have the unknown state to teleport:

$$|\phi\rangle_U = \alpha\,|0\rangle + \beta\,|1\rangle\,, \tag{6}$$

and three entangled photons handed to Alice, Bob and Charlie respectively:

$$|\xi\rangle_{ABC} = \frac{1}{2}(\,|000\rangle + |110\rangle + |011\rangle + |101\rangle). \tag{7}$$

If the supervisor Charlie allows the transmission between Alice and Bob, he would measure his qubit and send the classical one-bit information to the other two persons, thus giving two possible overall joint states for the four qubits involved:

$$\begin{aligned}|\psi\rangle_{UAB1} = \frac{1}{2}[\,&|\Phi^+\rangle_{UA}\,(\alpha\,|0\rangle + \beta\,|1\rangle)_B \\ &+ |\Psi^+\rangle_{UA}\,(\alpha\,|1\rangle + \beta\,|0\rangle)_B \\ &+ |\Phi^-\rangle_{UA}\,(\alpha\,|0\rangle - \beta\,|1\rangle)_B \\ &+ |\Psi^-\rangle_{UA}\,(\alpha\,|1\rangle - \beta\,|0\rangle)_B]\end{aligned} \tag{8}$$

$$\begin{aligned}|\psi\rangle_{UAB2} = \frac{1}{2}[\,&|\Phi^+\rangle_{UA}\,(\alpha\,|1\rangle + \beta\,|0\rangle)_B \\ &+ |\Psi^+\rangle_{UA}\,(\alpha\,|0\rangle + \beta\,|1\rangle)_B \\ &+ |\Phi^-\rangle_{UA}\,(\alpha\,|1\rangle - \beta\,|0\rangle)_B \\ &+ |\Psi^-\rangle_{UA}\,(\alpha\,|0\rangle - \beta\,|1\rangle)_B].\end{aligned} \tag{9}$$

Alice then performs a Bell-state measurement on her two qubits U and A, and sends the two-bit classical information to Bob. Note that we now have eight possible outcomes at Bob's side due to the participation of Charlie, leaving eight quantum gate combinations: $I, X, Z, XZ, X, XX(I), XZ$ and $XXZ(Z)$. After utilising the three bits of information Bob can finally recover the initial unknown state $|\phi\rangle_U$.

With this controlled quantum teleportation those authors were able to implement a controlled quantum direct communication protocol. As the term direct communication suggests, this kind of communication does not require secret key distribution, instead Alice encode her message directly into quantum states $|+\rangle$ and $|-\rangle$ i.e. $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. The most obvious benefit of this protocol is that the information sent by Alice is never revealed to a third-party, eliminating the potential threat of a suspicious server at Charlie's side. As the paper states, this protocol is said to be completely secure as long as a perfect quantum channel is provided. There was a similar method published in 2006 which uses W state for the three distributed entangled qubits [20]. W state is said to be infrangible in the case of qubit loss, compared to GHZ state where the remaining two qubits will be no longer in entangled state when one in three of them is lost. The so-called symmetric W state is written as:

$$|W\rangle_{123} = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)_{123} \tag{10}$$

with the orthogonal basis of $|0\rangle$ and $|1\rangle$.

The work in [17] explained a more complex QSDC scheme which promotes the application of a QSDC network, where the sender and receiver communicate via a server to enhance security. Fig. 3 shows a subnet of this kind of network in a loop topological structure. The protocol has a thorough security check of communication channels, mainly focuses on eavesdropping detection. The checking process is explained as following:

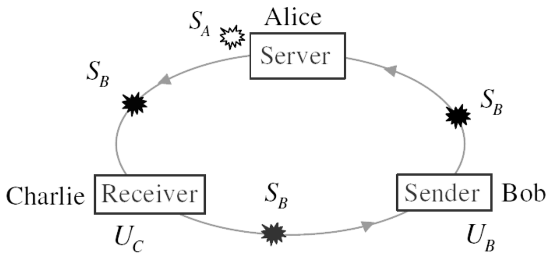


Fig. 3 QSDC network subsystem communication protocol. Source: [17]

1. The server Alice prepares a sequence of entangled qubit pairs, and for every pair take one of the qubits, store them in a sequence, say S_A, where the other side of the pairs are stored in S_B. Alice then send S_B to the receiver Charlie and keeps S_A.
2. Charlie conducts the first security check with Alice using S_B to detect eavesdropping by measuring the error rate.
3. Charlie then encrypt S_B using some local unitary operations U_C, and send to the sender Bob for the second security check.
4. If the noise rate is acceptable, Bob encodes his message into S_B with operations U_B and send to the server Alice.
5. Joint Bell-state measurement on S_A and S_B is performed by Alice, and the classical bit result is shared with Charlie. With the given data and information of U_C, Charlie is able to calculate the operations Bob performed, which was U_B, thus obtaining the original message.

Within the process any unacceptable noise rate will result in a complete restart of the protocol, hence quantum error correction codes are needed to mitigate the channel noise.

The authors in [17] also states that “high-dimensional quantum communication protocols provide better security than that obtainable with two-dimensional quantum systems," because “the two authorized users can use more than two sets of unbiased measuring bases to check eavesdropping". According to their theoretical formula the error rate caused by eavesdropping will be doubled in their subsystem compared to that of BB84 quantum key distribution protocol. Furthermore, since the receiver uses local operations U_C to encode S_B, Alice cannot obtain the original message sent by Bob, avoiding the latent dishonest server. It is also worth mentioning that this method was developed specifically for quantum communication with superdense coding [31], however as we can see, in principle it could be adapted to quantum teleportation with little modification.

5. Quantum Teleportation Simulation

5.1 Quantum gates and quantum circuits

In this section we briefly introduce how qubits are manipulated in theory, which leads to the concepts of quantum gates and circuits [6]. A qubit can be physically transformed by applying unitary matrices (or operators). Such transformations are synthesised with *Pauli matrices* and are all reversible. The term *reversible* means the output of an operation can be reversed to obtain its input data, which is obviously the opposite of what we know in practical electric circuits. According to those operators we introduce new types of gates I, X, Z, H and $CNOT$ that are used in quantum teleportation.

Quantum gates are mathematically represented in Pauli matrices:

$$\begin{aligned} I=\begin{pmatrix}1&0\\0&1\end{pmatrix}\ X=\begin{pmatrix}0&1\\1&0\end{pmatrix}\ Z=\begin{pmatrix}1&0\\0&-1\end{pmatrix} \\ H=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\equiv\frac{1}{\sqrt{2}}(X+Z) \end{aligned} \tag{11}$$

Consider we have a qubit in the form $|\phi\rangle=\alpha|0\rangle+\beta|1\rangle$, by applying those gates we have:

$$I|\phi\rangle=\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}\alpha\\\beta\end{pmatrix}\equiv\alpha|0\rangle+\beta|1\rangle \tag{12}$$

$$X|\phi\rangle=\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}\alpha\\\beta\end{pmatrix}\equiv\beta|0\rangle+\alpha|1\rangle \tag{13}$$

$$Z|\phi\rangle=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}\alpha\\\beta\end{pmatrix}\equiv\alpha|0\rangle-\beta|1\rangle \tag{14}$$

$$\begin{aligned} H|\phi\rangle&=\frac{1}{\sqrt{2}}(X|\phi\rangle+Z|\phi\rangle)\\ &=\frac{1}{\sqrt{2}}[\alpha(|0\rangle+|1\rangle)+\beta(|0\rangle-|1\rangle)]\\ &\equiv\alpha|+\rangle+\beta|-\rangle \end{aligned} \tag{15}$$

We can easily see that I gate preserves the input, and X gate swaps the amplitudes. Z gate applies a π phase shift to the two states, due to the result can also be represented as $\alpha|0\rangle+e^{i\pi}\beta|1\rangle$. The name of H gate is Hadamard matrix gate, where its function is to transform any qubit into a superposition state $\alpha|+\rangle+\beta|-\rangle$.

Controlled-NOT gate, or CNOT, is more special since it requires two inputs and gives two outputs. The first qubit, called the *control qubit*, decides whether the other *target qubit* is flipped (or amplitudes swapped if in superposition). For the matrices and complete truth table refer to pages 316 to 318 in [6]. Derived from

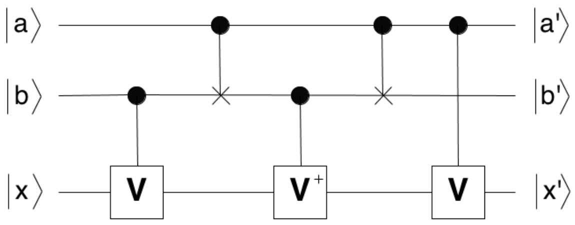


Fig. 4 Quantum circuit of a CCU gate. Source [6].

the idea of CNOT, a new category called controlled-U gates were invented. U can be any quantum gate with 2×2 Pauli matrix, such as X, Z and H. The control qubit decides whether the U gate is applied to the target qubit.

Quantum circuits model processes in quantum computing by using a sequence of quantum gates. Fig. 4 is an example circuit that functions as a controlled-controlled-U (CCU) gate [6], assuming we have a V gate such that $VV^{+} = I$ and $V^2 = U$. The dot-cross notation stands for CNOT gate, with dot represents the control bit. In a similar way a dot followed by a gate stands for a controlled-U gate.

5.2 Simulation program

The simulation is realised with a C++ program that uses Dlib [32] for quantum computing simulation. Fig. 5 shows the overall quantum circuit the program simulates.

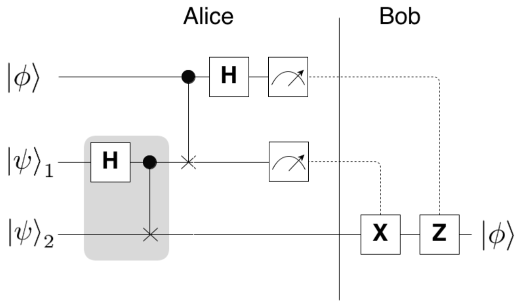


Fig. 5 Quantum teleportation full circuit

The shaded area in the graph is the apparatus that entangles $|\psi\rangle_{12}$, and the other parts involves $|\phi\rangle$ and $|\psi\rangle_1$ function as a Bell-state measurement, which also is an entanglement process. The graphical representation in Fig. 6 stands for a qubit measurement process. It gives a classical bit result, while the qubit is disturbed (collapse into a pure state) as a consequence. After the measurement, two classical bits are sent to the lane where controls the qubit $|\psi\rangle_2$ at Bob's side. We can also regard the two gates as controlled-X and controlled-Z gates, which transform $|\psi\rangle_2$ according to the measurement outcomes.

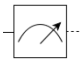


Fig. 6 Qubit measurement

The program simulates quantum teleportation by strictly following the circuit. A random qubit $|\phi\rangle$ is firstly generated, followed by adding two empty qubits $|0\rangle$ that stand for $|\psi\rangle_{12}$. Extra care was taken while generating the random qubit, since we had to preserve the axiom $|\alpha|^2 + |\beta|^2 = 1$. This was solved by firstly generate a random complex number with length less than one then calculate a corresponding β. Because we only have the information of β's length, the angle of β is a random degree in the range of 0° to 360°. The program was also adapted to have high efficiency when it is ran repetitively, which is an important property since there are potential usages of quantum teleportation in some other larger quantum computing simulation system. In order to function well with fast repetitive execution the random seed was set to microsecond of current time.

6. Simulation Results

In this section we present outputs of the program and validate its correctness. Before went into repetitive executions the correctness of individual steps was checked. Fig. 7 shows the program output of three possible outcomes of the Bell-state measurement. What we print is the state vector of qubits in a quantum register (i.e. the tensor product of all qubits. Refer to page 327 in [6] for details of tensor product in qubits). For instance for the first two lines of output, which is the random qubit, the two complex numbers stand for the values of α and β in $|\phi\rangle = \alpha|0\rangle + \beta|1\rangle$. In a similar way, after appending the two empty qubits $|0\rangle$ to the random qubit we have:

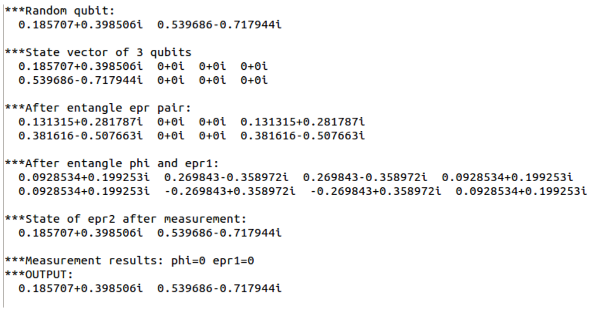
$$\begin{aligned}|\phi\rangle|\psi\rangle_{12} =& a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle \\ &+ e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle,\end{aligned} \tag{16}$$

and we can see in the outputs only a and e have values other than 0, which is correct. After the Bell-state measurement $|\phi\rangle$ and $|\psi\rangle_1$ are removed from the quantum register, thus we can obtain only the state vector of $|\psi\rangle_2$. The final output was given after applying the corresponding quantum gates, and there we can see the qubit amplitudes of $|\psi\rangle_2$ become exactly the same with that of the initial random qubit. This concludes our program flow check (white-box testing).

We then performed a reliability check of the program, regarding it as a module in some larger system (black-box testing). The module was left to run in a large number of times and the state vectors of the random qubit and output are compared programmatically. Fig. 8 shows outputs from the test. The result told that the program is able to cope with fast repetitive executions and provide reliable output qubits with 100% success rate.

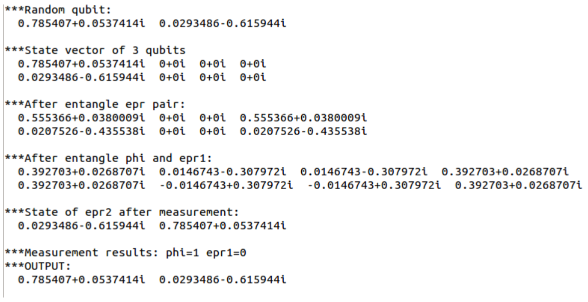
7. Future Works and Conclusions

By end of this project we found there were still moderate amount of topics that could be explored. The first



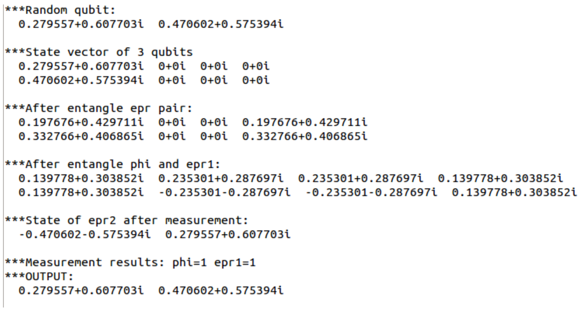
```
***Random qubit:
  0.185707+0.398506i  0.539686-0.717944i

***State vector of 3 qubits
  0.185707+0.398506i  0+0i  0+0i  0+0i
  0.539686-0.717944i  0+0i  0+0i  0+0i

***After entangle epr pair:
  0.131315+0.281787i  0+0i  0+0i  0.131315+0.281787i
  0.381616-0.507663i  0+0i  0+0i  0.381616-0.507663i

***After entangle phi and epr1:
  0.0928534+0.199253i  0.269843-0.358972i  0.269843-0.358972i  0.0928534+0.199253i
  0.0928534+0.199253i  -0.269843+0.358972i  -0.269843+0.358972i  0.0928534+0.199253i

***State of epr2 after measurement:
  0.185707+0.398506i  0.539686-0.717944i

***Measurement results: phi=0 epr1=0
***OUTPUT:
  0.185707+0.398506i  0.539686-0.717944i
```

(a) Result 1



```
***Random qubit:
  0.785407+0.0537414i  0.0293486-0.615944i

***State vector of 3 qubits
  0.785407+0.0537414i  0+0i  0+0i  0+0i
  0.0293486-0.615944i  0+0i  0+0i  0+0i

***After entangle epr pair:
  0.555366+0.0380009i  0+0i  0+0i  0.555366+0.0380009i
  0.0207526-0.435538i  0+0i  0+0i  0.0207526-0.435538i

***After entangle phi and epr1:
  0.392703+0.0268707i  0.0146743-0.307972i  0.0146743-0.307972i  0.392703+0.0268707i
  0.392703+0.0268707i  -0.0146743+0.307972i  -0.0146743+0.307972i  0.392703+0.0268707i

***State of epr2 after measurement:
  0.0293486-0.615944i  0.785407+0.0537414i

***Measurement results: phi=1 epr1=0
***OUTPUT:
  0.785407+0.0537414i  0.0293486-0.615944i
```

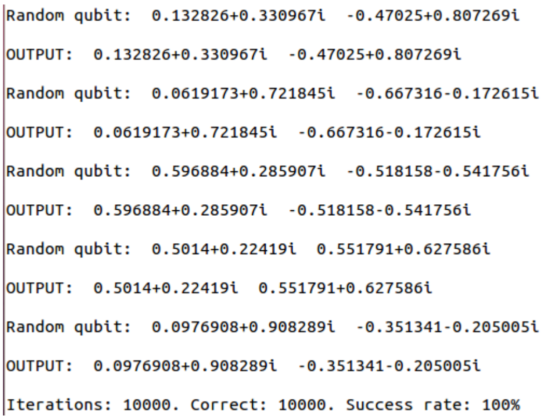
(b) Result 2



```
***Random qubit:
  0.279557+0.607703i  0.470602+0.575394i

***State vector of 3 qubits
  0.279557+0.607703i  0+0i  0+0i  0+0i
  0.470602+0.575394i  0+0i  0+0i  0+0i

***After entangle epr pair:
  0.197676+0.429711i  0+0i  0+0i  0.197676+0.429711i
  0.332766+0.406865i  0+0i  0+0i  0.332766+0.406865i

***After entangle phi and epr1:
  0.139778+0.303852i  0.235301+0.287697i  0.235301+0.287697i  0.139778+0.303852i
  0.139778+0.303852i  -0.235301-0.287697i  -0.235301-0.287697i  0.139778+0.303852i

***State of epr2 after measurement:
  -0.470602-0.575394i  0.279557+0.607703i

***Measurement results: phi=1 epr1=1
***OUTPUT:
  0.279557+0.607703i  0.470602+0.575394i
```

(c) Result 3

Fig. 7 Quantum teleportation simulation with different measurement results.



```
Random qubit:  0.132826+0.330967i  -0.47025+0.807269i

OUTPUT:  0.132826+0.330967i  -0.47025+0.807269i

Random qubit:  0.0619173+0.721845i  -0.667316-0.172615i

OUTPUT:  0.0619173+0.721845i  -0.667316-0.172615i

Random qubit:  0.596884+0.285907i  -0.518158-0.541756i

OUTPUT:  0.596884+0.285907i  -0.518158-0.541756i

Random qubit:  0.5014+0.22419i  0.551791+0.627586i

OUTPUT:  0.5014+0.22419i  0.551791+0.627586i

Random qubit:  0.0976908+0.908289i  -0.351341-0.205005i

OUTPUT:  0.0976908+0.908289i  -0.351341-0.205005i

Iterations: 10000. Correct: 10000. Success rate: 100%
```

Fig. 8 Success rate of teleporting 10000 qubits, with the last a few checks shown in the picture.

improvement we would do if the schedule were more relaxing is to introduce noise to both of the classical channel and quantum channel, which should bring further interesting results about error ratio in the output qubits. This may also introduce the need of quantum error correction algorithms, such as the Shor code [33]. A further perspective could be the implementation of a larger quantum computing simulation system. In that case the current quantum teleportation module could be used as a data transmission protocol, or if with more effort, a quantum network based on teleportation is also possible, which might again refer to [1] and [17].

In conclusion, this report functions as a step stone for fresh computer scientists to get start in the world of quantum information technology, more specifically, in the subject of quantum teleportation (QT). We introduced and defined QT in details, summarised its possible applications and limitations. The history and many current achievements were concluded and explained technically. Finally we presented a robust simulation program that realises this cutting-edge technology with own hand.

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