Near-Capacity Irregular Convolutional Coded Cooperative Differential Linear Dispersion Codes Using Multiple-Symbol Differential Detection

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***Abstract*—We propose a novel near-capacity Multiple-Symbol Differential Decoding (MSDD) aided cooperative Differential Linear Dispersion Code (DLDC) scheme, which exhibits a high grade of system design flexibility in terms of the choice of activated relays and the DLDC's rate allocation. More specifically, the system has the freedom to activate a range of DLDCs depending on both the number of relays available in the network, as well as on their position, throughput and complexity considerations.**

***Index Terms*—Cooperative communications, differential linear dispersion codes, irregular convolutional codes, relay selection.**

I. Introduction

COOPERATIVE diversity was proposed in [1], where the single-element Mobile Stations (MSs) share their antennas to form a Virtual Antenna Array (VAA) and as a benefit, they typically experience uncorrelated fading. However, it becomes unrealistic for the relays and the destination to estimate the channel of all the VAA links. To avoid the potentially excessive complexity of coherent MIMO detection, non-coherently detected DPSK may be used in each of the single-antenna links. Accordingly, Differential STBCs (DSTBCs) could be found in [2]. Furthermore, a Differential Linear Dispersion Code (DLDC) based on the so-called Cayley transform was proposed in [3] in order to strike a flexible trade-off between the achievable throughput and diversity gain.

In the absense of channel estimation, Conventional Differential Detection (CDD) generally suffers from a 3 dB performance penalty, provided that the Doppler frequency is not excessive, while upon increasing the Doppler frequency a pronounced irreducible error floor is formed. Multiple-Symbol Differential Detection (MSDD) was proposed in [4] in order to reduce the performance discrepancy. The MSDD observes N_w consecutive received symbols and makes a joint decision based on (N_w-1) information symbols. The price paid is that the complexity imposed increases exponentially with N_w. To mitigate the complexity, Multiple-Symbol Differential Sphere Decoding (MSDSD) was proposed for Differential Space-Time Modulation (DSTM) including DSTBCs as well as DLDCs in [5]. As a further advance, a soft-output MSDSD designed for DPSK was proposed in [6] for the sake of turbo detection. A novel MSDSD aided cooperative Amplify-and-Forward (AF) design was proposed in [7], where a low BER can only be achieved in the high-SNR region.

Against this background, the novel contributions of this letter are: 1) we first propose a soft-output MSDSD for the DSTBC/DLDC scheme, so that MSDSD may be applied for turbo detection in cooperative Decode-and-Forward (DF) systems. 2) We also propose a near-capacity MSDD/MSDSD aided cooperative DLDC scheme, which allows flexible relay selection and cooperative rate allocation.

The following notation is used throughout the letter. A DLDC is described by the nomenclature of $DLDC(MNTQ)$, where M and N indicate the number of transmit and receive antennas, while T and Q denote the number of channel uses and the number of transmitted symbols per block, respectively. Furthermore, N_w refers to the window length of the MSDD/MSDSD.

II. System Overview

The schematic of the cooperative system considered is shown in Fig. 1, where an Up-Link (UL) scenario is considered. Both the source and the relays are assumed to be single-antenna-aided Mobile Stations (MSs), while the destination is assumed to be a Base Station (BS) having two antennas. As shown in [8], according to free space path loss, the Reduced-Pathloss-Related (RPLR) power gain of the source-relay (SR) link with respect to the Source-Destination (SD) link G_{SR} and the RPLR power gain of the Relay-Destination (RD) link with respect to the SD link G_{RD} have the relationship of

$$\frac{1}{\sqrt{G_{SR}}}+\frac{1}{\sqrt{G_{RD}}}=1 \quad (1)$$

and the terminology of the equivalent transmit SNR at the source was introduced in [8] as

$$\begin{aligned}\mathrm{SNR}_t &= \mathrm{SNR}_{SR}-10\log_{10}(G_{SR}) \quad \mathrm{dB}\\ &= \mathrm{SNR}_{RD}-10\log_{10}(G_{RD}) \quad \mathrm{dB} \quad (2)\end{aligned}$$

In order to achieve an infinitesimally low BER, turbo detection may be employed at both the relays and at the destination. Similar to the classic Recursive Systematic Convolutional (RSC) codes, the differential encoder of the DPSK/DSTM has a recursive structure. Hence theoretically a free distance of $d=2$ may be achieved by a combined RSC decoder and a MSDD/MSDSD having a detection window size as long as the encoding frame length. However, the detection window size of MSDSD is severely limited because of its complexity. As an alternative mean, a Unity Rate Code (URC) may be employed as seen in Fig. 2. If the relays and the destination are able to afford

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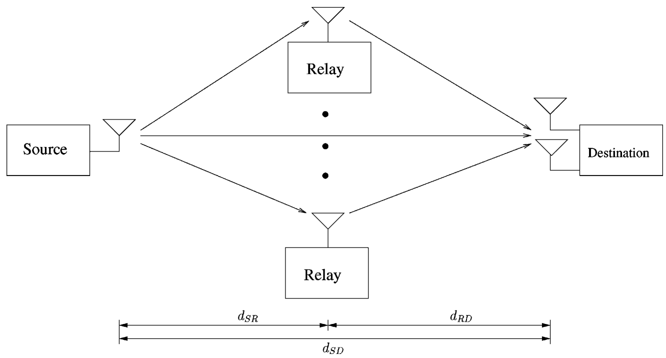

Fig. 1. Block diagram of a relay-aided uplink system.

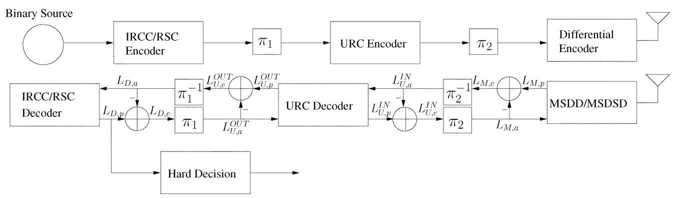

Fig. 2. Schematic of the three-component serial-concatenated encoder at the source and relays, and the corresponding three-stage turbo detection at the relays and the destination.

the IRregular Convolutional Code (IRCC) decoding complexity, then near-capacity performance may be achieved.

In the first transmission period, the source transmits IRCC/RSC coded as well as URC precoded DPSK symbols to both the relays and to the destination. At the relays, hard decisions are made after the three-stage turbo MSDD/MSDSD. Then the IRCC/RSC and URC re-encoded DLDC symbols are transmitted by the relays during the second transmission period. At the destination, the same turbo detection process is carried out for the coded DLDC symbols received from the relays. The overall throughput of the proposed cooperative scheme is given by

$$R = \frac{R_S \cdot b_S}{1 + \frac{R_S \cdot b_S}{R_R \cdot \left(\frac{Q \cdot b_R}{T}\right)}} \tag{3}$$

where R_S and R_R are the rates of the IRCC/RSC employed at the source and at the relays, respectively. The variables b_S and b_R denote the number of modulated bits per symbol of the DPSK modulation scheme employed at the source node, and that of the DLDC employed at the relay node.

The choice of the DLDC parameters M and N depends on how many relays are available in the network as well as on how many UL receive antennas are used at the BS. The DLDC throughput specified by $Q \cdot b_R/T$ is determined by the position of the relays, which specifies RPLR power gains G_{SR} and G_{RD}. Our proposed DLDC selection designed for the cooperative DF scheme is detailed in Section IV.

III. Soft-Decision Aided MSDSD Decoder Designed for DSTM

Differential encoding designed for DSTM schemes may be formulated in a way similar to classic DPSK, yielding

$$\mathbf{S}_n = \begin{cases} \mathbf{S}_1 & n = 1 \\ \mathbf{X}_{n-1}\mathbf{S}_{n-1} & n > 1 \end{cases} \tag{4}$$

where the $(T \times T)$-element unitary matrix $\mathbf{X}_n$ carries the source information, while the transmission matrix $\mathbf{S}_n$ has a size of $(T \times M)$.

We assume that the Rayleigh fading envelope and phase remain constant over T channel uses and that the received signal is contaminated by Additive White Gaussian Noise (AWGN), hence the received signal may be modelled as

$$\mathbf{Y}_n = \mathbf{S}_n\mathbf{H}_n + \mathbf{V}_n \tag{5}$$

where the matrix $\mathbf{Y}_n$ has a size of $(T \times N)$. The AWGN matrix $\mathbf{V}_n$ has the same size, a zero mean and a variance of N_0 for each dimension. The channel matrix $\mathbf{H}_n$ is of size $(M \times N)$, and it is generated according to Clarke's fading model.

As mentioned before, the MSDD/MSDSD observes N_w consecutive received signal blocks $\{\mathbf{Y}_n\}_{n=1}^{N_w-1}$ and makes a joint decision based on $(N_w - 1)$ consecutive information blocks $\{\mathbf{X}_n\}_{n=1}^{N_w-1}$. The hard-output MSDSD designed for the DSTM of [5] should be modified in order to be used in turbo detection. Similarly to the soft-output DPSK MSDSD of [6], the Sphere Decoder (SD) aims to find the optimum $(N_w - 1)$ blocks $\{\mathbf{X}_n\}_{n=1}^{N_w-1}$ that leads to the minimum Euclidean distance, which is formulated as

$$\sum_{i=1}^{N_w-1} \left\| \sum_{j=i}^{N_w} l_{ji}\mathbf{S}_j^H\mathbf{Y}_j \right\|^2 - \log\left(\Pr(\{\mathbf{X}_n\}_{n=1}^{N_w-1})\right) \leq R^2 \tag{6}$$

where R denotes the decoding sphere radius, which is minimized by the SD. The coefficient l_{ji} in (6) represents the predictor coefficients hosted by the corresponding elements in the lower triangular matrix $\mathbf{L}$ in [6, eq. (6)]. The *a priori* probability in (6) may be calculated by the product of the *a priori* individual information block probabilities according to the *a priori* LLRs and the corresponding binary bit combinations.

The most recent transmission matrix $\mathbf{S}_{N_w}$ is a common multiplier for all the transmission matrices. Hence we introduce the accumulated information matrix in order to eliminate the influence of $\mathbf{S}_{N_w}$, which may be formulated as

$$\mathbf{A}_n = \mathbf{S}_n\mathbf{S}_{N_w}^H = \begin{cases} \prod_{i=n}^{N_w-1} \mathbf{X}_i^H & 1 \leqslant n < N_w \\ \mathbf{I_T} & n = N_w \end{cases}. \tag{7}$$

Let us now define the Partial Euclidean Distance (PED) component seen in (6) as

$$\begin{aligned} d_i^2 &= \sum_{t=i}^{N_w-1} \left(\left\| \sum_{j=t}^{N_w} l_{jt}\mathbf{S}_j^H\mathbf{Y}_j \right\|^2 - \log(\Pr\{\mathbf{X}_t\}) \right) \\ &= d_{i+1}^2 + \left\| l_{ii}\mathbf{X}_i\mathbf{Y}_i + \mathbf{A}_{i+1}\left(\sum_{j=i+1}^{N_w} l_{ji}\mathbf{A}_j^H\mathbf{Y}_j \right) \right\|^2 \\ &\quad - \log(\Pr\{\mathbf{X}_i\}) \end{aligned} \tag{8}$$

with $i = 1, 2, \ldots, (N_w - 1)$, and it lies within the decoding sphere. Each time the MSDSD performs sphere decoding, the $(N_w - 1)$ blocks $\{\mathbf{X}_n\}_{n=1}^{N_w-1}$ giving the minimum decoding sphere radius R are found. Therefore the MSDSD constitutes the Max-Log-MAP approximation of the ML-MSDD, where only two optimum combinations are taken into account, which is formulated as

$$\log\left(\frac{\Pr\{b_k = 1 \mid \mathbf{Y}\}}{\Pr\{b_k = 0 \mid \mathbf{Y}\}}\right) \approx d_{MAP}^{b_k=1} - d_{MAP}^{b_k=0} \tag{9}$$

TABLE I
LOOKUP TABLE SUMMARIZING THE SNR_{RD} REQUIRED FOR ACHIEVING AN INFINITESIMALLY LOW BER FOR THE THREE-STAGE TURBO DETECTED IRCC-URC-DLDC(323Q)/DG3 MSDD SCHEME, WHEN $f_d = 0.03$.

URC-DLDC(3231)	$\mathrm{SNR_{RD}}$ for $N_w = 2$	$\mathrm{SNR_{RD}}$ for $N_w = 4$	$\mathrm{SNR_{RD}}$ for $N_w = 6$
1 inner iteration	-3.6 dB	-4.4 dB	-4.7 dB
2 inner iterations	-3.6 dB	-5.2 dB	-5.8 dB
URC-DLDC(3232)	$\mathrm{SNR_{RD}}$ for $N_w = 2$	$\mathrm{SNR_{RD}}$ for $N_w = 4$	$\mathrm{SNR_{RD}}$ for $N_w = 6$
1 inner iteration	-1.2 dB	-2.1 dB	-2.4 dB
2 inner iterations	-1.6 dB	**-3.2 dB**	-3.7 dB
URC-DLDC(3233)	$\mathrm{SNR_{RD}}$ for $N_w = 2$	$\mathrm{SNR_{RD}}$ for $N_w = 4$	$\mathrm{SNR_{RD}}$ for $N_w = 6$
1 inner iteration	0.4 dB	-0.5 dB	-0.8 dB
2 inner iterations	-0.2 dB	-1.7 dB	-2.2 dB
URC-DG3 with 2Rxs	$\mathrm{SNR_{RD}}$ for $N_w = 2$	$\mathrm{SNR_{RD}}$ for $N_w = 4$	$\mathrm{SNR_{RD}}$ for $N_w = 6$
1 inner iteration	-0.1 dB	-0.7 dB	-0.9 dB
2 inner iterations	-0.1 dB	-1.7 dB	-2.3 dB

where $d_{MAP}^{b_k=1}$ and $d_{MAP}^{b_k=0}$ denote the minimum Euclidean distance, when b_k is fixed to 1 and 0, respectively. As a result, the reliability of the LLRs is degraded. Therefore, similar to the soft-decision aided MSDSD designed for DPSK, the observation windows are shifted only by one block at a time, while only the LLRs of the central data block are calculated each time, and N_w has to be an even number.

IV. DESIGN AND PERFORMANCE

In this section, we propose a new relay selection and rate allocation design for the near-capacity MSDD aided cooperative DLDC scheme, followed by our simulation results.

Based on EXIT chart estimation and Monte-Carlo simulation, the SNR_{RD} required for achieving an infinitesimally low BER at the destination is summarized in Table I. It has been widely exploited [9] that a vanishingly low BER is achievable, when an open EXIT chart tunnel is formed between the EXIT curves of the inner and outer code. Furthermore, according to the area properties of EXIT charts [9], the area under the EXIT curve of the inner decoder is appropriately equal to the channel capacity attained. Therefore, a near-zero area in the EXIT open tunnel implies a near-capacity performance. However, in practice not all received frames can be error-freely decoded at the convergence SNR estimated by the EXIT chart, since the EXIT chart is only accurate for an infinite interleaver length. In practical Monte-Carlo simulations using a finite interleaver-length, normally about 0.5 dB higher SNR is required. We assume that there are three available relays, hence the class of DLDC(32TQ) and DG3 are of interest. Without loss of generality, we consider the DSTM scheme employing BPSK, i.e., we have ($b_R = 1$), since increasing the number of Q is preferred compared to employing a higher-level modulation scheme. Furthermore, since the diversity gain of DLDC is determined by $N \min\{M, T\}$ [3], a setting of $T = M$ is fixed.

Table I demonstrates that a substantial design flexibility may be provided by the rich set of DSTM schemes having different rates. It was demonstrated in [10] that having a flexible code rate allocation was beneficial for the cooperative DF schemes. In this letter, we propose to allocate the code-rate by appropriately choosing the different modulation schemes. The design procedures proposed for our MSDD aided cooperative DLDC scheme are as follows.

1) We first determine the modulation scheme employed at the source node based on the throughput consideration and the affordable complexity for the relays, then the SNR_{SR} required at the relays to avoid error propagation is determined.
2) The BS chooses the parameters M and N for DSTM according to the number of available relays, and then sets up the corresponding lookup table. Table I represents the scenario of $M = 3$ as well as $N = 2$ for $f_d = 0.03$.
3) The power gains G_{SR} and G_{RD} are determined by the positions of the activated relays. Then the SNR_{RD} required at the destination may be calculated by $\mathrm{SNR}_{RD} = \mathrm{SNR}_{SR} - 10\log_{10}(G_{SR}) + 10\log_{10}(G_{RD})$.
4) Finally, the BS find a suitable code rate for the DLDC in the lookup table using the estimated SNR_{RD}.

It can be seen that the closer the relays approach the source node, the higher G_{SR} becomes compared to G_{RD}, which leads to a lower SNR_{RD} required at the destination. As a result, according to Table I, a lower DLDC code-rate has to be chosen for the weak RD link. By contrast, a higher-rate DLDC should be selected in the opposite situation.

We now present a design example. The modulation scheme of the SR link is first fixed to be DQPSK, and the MSDD with $N_w = 4$ is employed at the relays. The EXIT chart and decoding trajectory recorded for the SR links are shown in Fig. 3, where our 36-component IRCC [11] has the weighting coefficients of

$$[\alpha_1, \ldots, \alpha_{36}] = [0, 0, 0, 0, 0, 0.29678, 0.284534, 0.0698805, 0, 0, 0.0470204, 0.161304, 0, 0, 0.0864003, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.0541135].$$

The bit interleaver length is set to 10^6. It can be seen in Fig. 3 that $\mathrm{SNR}_{SR} = 5.7$ dB is required for achieving a vanishingly low BER at the relays. If the relays cannot afford the complexity of decoding the 36-component IRCC and/or using MSDD, then a single-component RSC as well as the MSDSD designed in Section III may be employed, and the bit interleaver length may be reduced to 10^4. It can be seen in Fig. 3 that employing a single-component RSC and MSDSD requires a 0.8 dB higher SNR at the relays.

As an example, we assume that there are three available relays in the network, which are located in a position, where $G_{SR} - G_{RD} = 9$ dB. According to the RPLR power gains relationship of (1), we have $G_{SR} \approx 11.63$ dB and $G_{RD} \approx 2.63$ dB, which requires $\mathrm{SNR}_{RD} = -3.3$ dB at the destination. Therefore, based on row x and column y of Table I, the DLDC(3232) using $N_w = 4$ for MSDD and two inner iterations within the URC-MSDD composite decoder may be selected, as indicated by the bold entry in Table I. As a result, according to (3), the overall throughput of the cooperative scheme is 0.25. The EXIT chart for the RD links is similar to the SR links of Fig. 3, and the IRCC employed for the selected modulation scheme has the weighting coefficients of

$$[\alpha_1, \ldots, \alpha_{36}] = [0, 0, 0, 0.310327, 0, 0, 0.113319, 0, 0.133812, 0, 0, 0.149363, 0, 0, 0.0922107, 0, 0.070601, 0, 0, 0, 0, 0, 0.0755869, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.0548512].$$

The BER performance evaluated at the destination is portrayed in Fig. 4. The maximum achievable rates indicated in Fig. 4 are calculated based on the area property of the EXIT chart [9]. It is shown in Fig. 4 that as a benefit of the cooperative

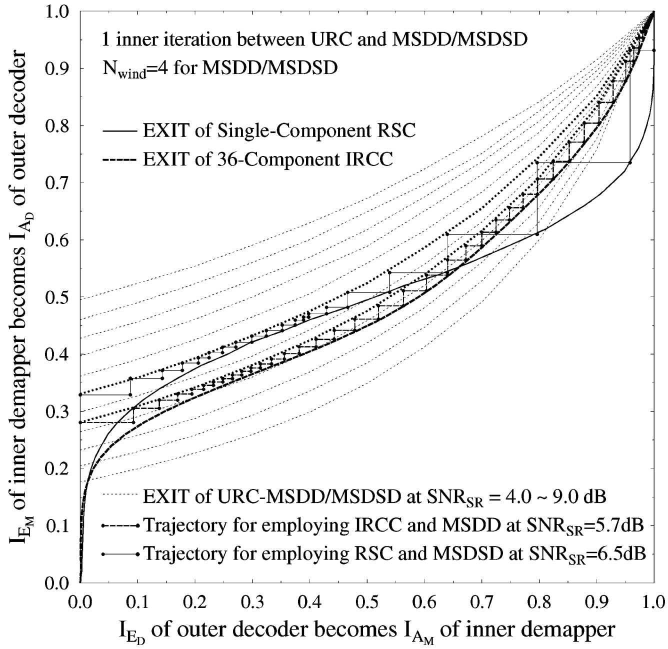

Fig. 3. EXIT chart of the three-stage turbo detected DQPSK scheme of Fig. 2 for the SR links, when $f_d = 0.03$. The 36-component IRCC having weighting coefficients and the MSDD with $N_w = 4$ are adopted, as well as a single inner iteration within the URC-MSDD composite decoder and a interleaver length of 10^6 are employed. The case of employing RSC and MSDSD with a interleaver length of 10^4 is also drawn.

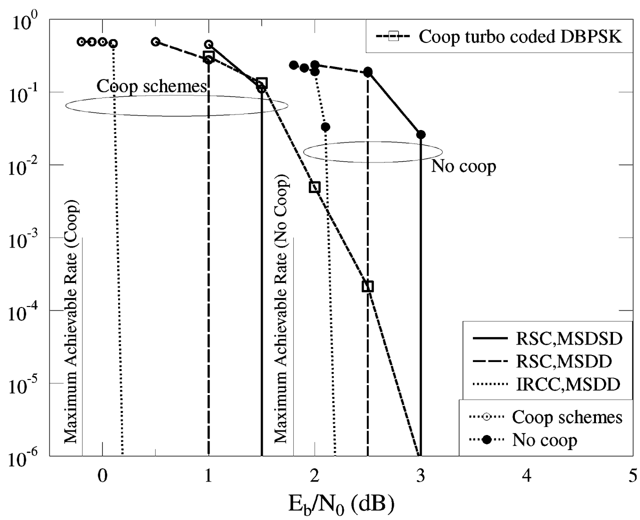

Fig. 4. BER performance of the proposed cooperative DLDC employing IRCC/RSC and MSDD/MSDSD, in comparison with their non-cooperative counterparts, for $f_d = 0.03$. A MSDSD aided cooperative turbo-coded DBPSK scheme is also drawn as a benchmark.

diversity, the proposed near-capacity cooperative scheme provides a 2.0 dB performance improvement compared to the conventional direct transmission operating without relaying. Furthermore, the low complexity cooperative scheme employing the single-component RSC as well as MSDSD also outperforms its non-cooperative counterpart by 1.5 dB.

As another benchmark, a cooperative turbo-coded DBPSK scheme with the same system throughput of 0.25 and the same MSDSD window length of $N_w = 4$ is also portrayed in Fig. 4. The schematics of the cooperative turbo-coded DBPSK was proposed in [12], where the IRCC/RSC and URC blocks of Fig. 2 are replaced by a single half-rate turbo code. The number of inner iterations within the turbo decoder is set to 2, and the number of outer iterations between the turbo decoder and the MSDSD is also set to 2. Fig. 4 shows that the non-cooperative RSC and URC coded DQPSK scheme's performance is comparable to the cooperative turbo-coded DBPSK, and all the proposed cooperative schemes outperform the turbo coded cooperative scheme. This is because the three-stage turbo detection employed in our proposed scheme has an open tunnel leading to the (1.0, 1.0) point of the EXIT chart, which results in a sharp turbo-cliff.

Our proposed cooperative DLDC scheme becomes capable of outperforming its cooperative LDC coherent-detection-aided counterpart relying on realistic imperfect channel estimation, when the Gaussian-distributed Channel State Information (CSI) estimation noise becomes -7.5 dB for the SR links, and -1.0 dB for the RD links. Viewing this somewhat surprising fact from a slightly different angle, for the QPSK aided coherent cooperative LDC scheme to maintain at least the same performance as DQPSK MSDSD aided cooperative DLDC, the CSI estimation error should be limited to -8.5 dB for the SR links, and -2.0 dB for the RD links.

In conclusion, we have proposed a near-capacity MSDD aided cooperative DLDC scheme, relying on a flexible relay selection and rate allocation design. We have demonstrated that near-capacity performance may be achieved by the employment of our 36-component IRCC, and the proposed cooperative scheme performs better than the conventional direct transmission regime operating without relaying and it also outperforms its cooperative turbo-coded DBPSK counterpart, which has the same system throughput.

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