Superposition Coding Aided Bi-directional Relay Transmission Employing Iteratively Decoded Self-Concatenated Convolutional Codes

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***Abstract*— In this paper, we consider coding schemes designed for two nodes communicating with each other with the aid of a relay node, which receives information from the two nodes in the first time slot. At the relay node we combine a powerful Superposition Coding (SPC) scheme with Iteratively-Decoded Self-Concatenated Convolutional Codes (SECCC-ID), which exchange mutual information between each other. It is assumed that decoding errors may be encountered at the relay node. The relay node then broadcasts this information in the second time slot after re-encoding it, again, using a SECCC encoder. At the destination, an amalgamated SPC-SECCC block then detects and decodes the signal either with or without the aid of *a priori* information. Our simulation results demonstrate that the proposed scheme is capable of reliably operating at a low BER for transmission over both AWGN and uncorrelated Rayleigh fading channels. We compare the proposed scheme's performance to a direct transmission link between the two sources having the same throughput. Additionally, the SPC-SECCC system achieves a low BER even for realistic error-infested relaying.**

I. Introduction

In Superposition Coded (SPC) schemes the multiple nodes' information is code-multiplexed in order to generate the superimposed and appropriately rotated composite signal, which results in a high throughput [1]–[3]. Hence an outer channel-coded SPC-aided bi-directional relaying arrangement is proposed, which was considered in the context of a relay-aided scenario in [4], [5] and in a two-user cooperative scenario in [6], [7]. In [8] devised a SPC Aided Multiplexed Hybrid ARQ (M-HARQ) scheme, where M-HARQ jointly encodes the current new packet to be transmitted and any packets that are about to be retransmitted.

The philosophy of concatenated coding schemes was proposed by Forney in [9]. Turbo codes constitute a class of high-performance error correction codes (ECC) based on parallel concatenated convolutional codes (PCCC) of two or more constituent codes, which were developed in [10]. Serially concatenated convolutional codes (SCCC) [11] have been shown to yield a performance comparable, and in some cases superior, to turbo codes. Iteratively-Decoded Self-Concatenated Convolutional Codes (SECCC-ID) proposed by Benedetto *et al.* [12] constitute another attractive family of iterative detection aided schemes, which impose a low complexity, since they require a single encoder and a single decoder.

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The novel contribution of this paper is that *we propose an SPC aided bi-directional relaying scheme, which receives information from the two communicating mobiles in the first time slot and after detecting as well as decoding the information retransmits it to the corresponding destinations during the second time slot. In contrast to prior studies, the source-to-relay link is not assumed to be error free, which was the case in the Network Coding (NC) schemes of [6], [7]. In contrast to [2], the system proposed does not rely on temporal diversity, and it requires a lower number of time slots than the solution in [2]. Our performance results show that the relay-aided SPC arrangement requires a lower transmit power than the direct transmission of information between the two mobile users while maintaining the same throughput (η) and delay (τ), because both the relay-aided and direct scheme require two time slots for their communications.*

The rest of the paper is organized as follows. In Section II, we describe our system model and outline the architecture of the bi-directional SPC-SECCC scheme proposed. Furthermore, the iterative receiver structure is also discussed. Section III is dedicated to our performance evaluations. Finally, we conclude our discourse in Section IV.

II. System Description

A. Cooperation Model

The communication links seen in Fig 1 are subject to both free-space path loss as well as to short-term uncorrelated Rayleigh fading.

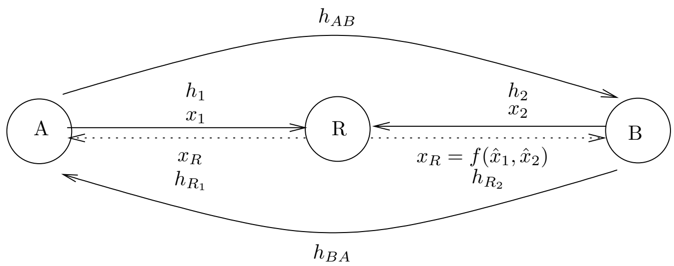

Fig. 1. Schematic of the bi-directional relay aided system.

Let S_{ab} denote the distance between nodes a and b. The path-loss between these nodes can be modelled by [13]:

$$P(ab) = K/S_{ab}^{\alpha} \ , \tag{1}$$

where K is a constant that depends on the environment and α is the path-loss exponent. For a free-space path-loss model we

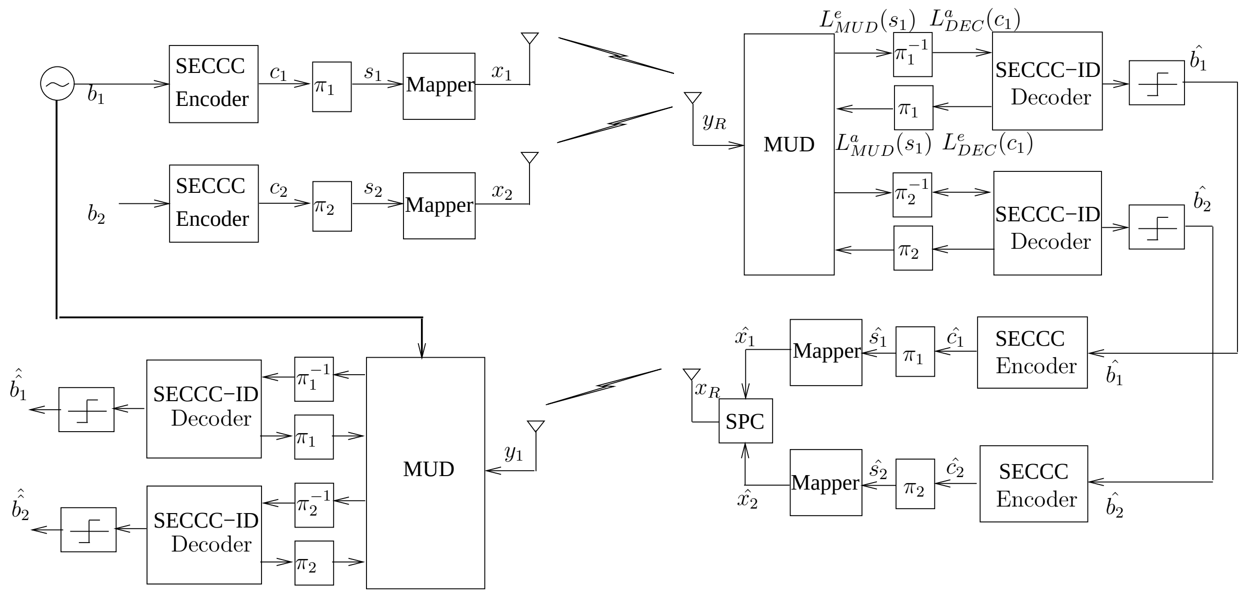

Fig. 2. SPC based SECCC-ID System (Phase I and II)

have $\alpha = 2$. The relationship between the energy E_{ar} received at the relay node and that of the destination node E_{ab} can be expressed as:

$$E_{ar} = \frac{P(ar)}{P(ab)} E_{ab} = G_{ar} E_{ab} \ , \tag{2}$$

where G_{ar} is the power-gain (or geometrical gain) [13] experienced by the source-relay link with respect to the source-destination link as a benefit of its reduced distance and path-loss, which can be computed as:

$$G_{ar} = \left(\frac{S_{ab}}{S_{ar}} \right)^2 \ . \tag{3}$$

Similarly, the power-gain of the relay-destination link with respect to the source-destination link can be formulated as:

$$G_{rb} = \left(\frac{S_{ab}}{S_{rb}} \right)^2 \ . \tag{4}$$

If $x_{a,j}$ is the jth symbol transmitted from node a, the average received Signal to Noise power Ratio (SNR) at node b is given by:

$$\text{SNR}_r = \frac{\text{E}\{G_{ab}\}\text{E}\{|h_{ab,j}|^2\}\text{E}\{|x_{a,j}|^2\}}{N_0} = \frac{G_{ab}}{N_0} \ , \tag{5}$$

where $\text{E}\{|h_{ab,j}|^2\} = 1$ and $\text{E}\{|x_{a,j}|^2\} = 1$. For ease of analysis, we define the ratio of the power transmitted from node a to the noise power encountered at the receiver of node b as:

$$\text{SNR}_t = \frac{\text{E}\{|x_{a,j}|^2\}}{N_0} = \frac{1}{N_0} \ , \tag{6}$$

which implies relating the noise and signal powers to each other at different points in space - therefore we refer to it as the equivalent SNRs. Hence, we have:

$$\begin{aligned} \text{SNR}_r &= \text{SNR}_t \ G_{ab} \ , \\ \gamma_r &= \gamma_t + 10\log_{10}(G_{ab}) \ [\text{dB}] \ , \end{aligned} \tag{7}$$

where $\gamma_r = 10\log_{10}(\text{SNR}_r)$ and $\gamma_t = 10\log_{10}(\text{SNR}_t)$. Therefore, we can achieve the desired SNR_r either by changing the transmit power or by selecting a relay at a different geographical location. In order to quantify SNR_r in terms of E_b/N_0, we have to consider the rate R of the SECCC encoder, hence we have $E_b/N_0 = \gamma - 10\log_{10}(R)$.

The bi-directional relaying arrangement relies on three nodes, A, B and R, as shown in Fig 1. Node A and B intend to communicate with each other with the aid of node R. This can be achieved using any of the cooperation strategies outlined in Table I.

Cooperation schemes	T_1	T_2	T_3	T_4
Conventional Relaying	A→R	R→B	B→R	R→A
Network Coding	A→R	B→R	R→A R→B	
SuperPosition Coding	A→R B→R	R→A R→B		

TABLE I

COMPARISON OF VARIOUS COOPERATION PHILOSOPHIES.

- Conventional relaying - Node A transmits x_1 to R in the first time slot (T_1), while R relays the message to Node B in the second time slot (T_2). Node B transmits x_2 to R in the third slot (T_3), while R relays the message to Node B in the fourth time slot (T_4).
- NC - Node A transmits x_1 to R in (T_1). Node B transmits x_2 to R in (T_2), while R relays the message to the intended destination in (T_3)[1].
- SPC - Only two transmission phases are required. Node A and B transmits x_1 and x_2 to R in (T_1) (Phase-I) and R relays the message to the intended destination in (T_2) (Phase-II).

B. Phase I - Source to Relay Transmission

The basic block diagram of the SPC-SECCC scheme is given in Fig 2, which relies on Phases I and II, representing two different time slots. The Phase-I transmission may be considered as a two-user UpLink (UL) multiple access scenario, where each user employs a powerful rate-R SECCC scheme and QPSK modulation, for their transmission to R, which

[1]It is possible to achieve the same in two time slots by using Physical-Layer Network Coding (PNC) [14] at the 'cost' of the additional complexity of a specially designed mapping scheme.

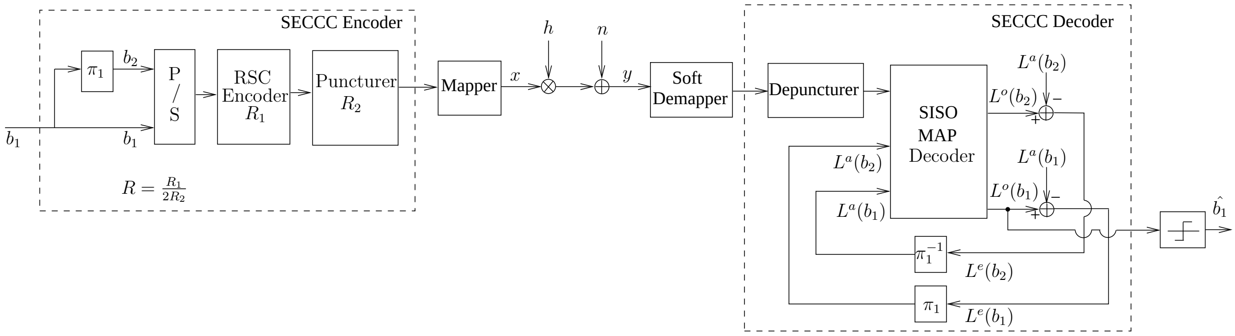

Fig. 3. Schematic of the SECCC encoder and decoder. The notation $L(b)$ denotes the Log-Likelihood Ratio (LLR) of bit b and the superscripts a, o and e denote the *a priori*, *a posteriori* and extrinsic nature of the LLR, respectively [15].

plays the role of a BS. The two UL streams are separated by a user-specific bit-interleaver, resulting the well-known Bit-interleaved Coded Modulation [16], [17]. The discrete-time system model may be written as:

$$y_R = h_1 x_1 + h_2 x_2 + n_R, \tag{8}$$

where y_R, x_1 and x_2 denote the received signal at the relay and the two transmitted signals emerge from sources A and B, respectively. Furthermore, h_1 and h_2 denote the corresponding fading coefficient between source A and the relay R as well as between source B and the relay R, while n_R denotes the complex-valued Additive White Gaussian Noise (AWGN).

1) SECCC Encoding at Source: At the source node we consider a rate $R = 1/3$ SECCC scheme combined with Quadrature Phase-Shift Keying (QPSK) modulation. Both AWGN and uncorrelated Rayleigh fading channel conditions are considered.

As shown in Fig 3, the input bit sequence $\{b_1\}$ of the self-concatenated encoder is interleaved for generating the bit sequence $\{b_2\}$. The resultant bit sequences are parallel-to-serial converted and then fed to the Recursive Systematic Convolutional (RSC) encoder, which employs the generator polynomial G=[13 15] expressed in octal format and having a rate of $R_1 = \frac{1}{2}$ as well as a memory of $\nu = 3$. Hence, for every bit input to the SECCC encoder there are four output bits of the RSC encoder. At the output of the encoder seen in Fig 3 there is an interleaver and then a rate $R_2 = \frac{3}{4}$ puncturer, which punctures (does not transmit) one bit out of four encoded bits. Hence, the overall code rate, R can be derived based on [18] as

$$R = \frac{R_1}{2 \times R_2} = \frac{1/2}{2\,(3/4)} = \frac{1}{3} \tag{9}$$

Puncturing is used in order to increase the achievable bandwidth efficiency η. Different codes have been designed in [19] by varying the rates R_1 and R_2. These bits are then mapped to a QPSK symbol as $x = \mu(c_1 c_0)$, where $\mu(.)$ is the bit-to-symbol mapping function. Hence the resultant bandwidth efficiency is given by $\eta = R \times \log_2(4) = 0.67$ bit/s/Hz, assuming a Nyquist roll-off-factor of α=0. The QPSK symbol x_s is then transmitted over the channel. The corresponding SECCC-ID decoder of Fig. 3 has been further explained in [15].

2) Multiuser Detection: A host of Multiuser Detection (MUD) schemes may be invoked, including the powerful but potentially complex Maximum Likelihood (ML) detection scheme, sphere decoding [20], etc. Here we opt for employing a low-complexity soft interference cancellation scheme [21].

Since a sufficiently long bit interleaver employed is capable of mitigating the correlation between consecutive symbols, we consider a particular symbol and aim for the detection of the jth source's symbol x_j, where Eq. (8) may be rewritten as

$$y_R = h_j x_j + \xi, \tag{10}$$

with ξ representing the residual interference plus noise. By approximating ξ as a joint Gaussian random vector, we can model the extrinsic symbol probability as:

$$\Pr^e(x_j = x) \quad \propto \quad \exp\left[-|y_R - \hat{\xi} - h_j x|^2/2V_\xi\right], \tag{11}$$

where $x \in \mathcal{A}$ is the particular realization drawn from the modulation alphabet $\mathcal{A}$. The estimated value of ξ and its variance V_ξ may be expressed as

$$\hat{\xi} \quad = \quad \sum_{i=1}^{2} h_i \hat{x}_i - h_j \hat{x}_j, \tag{12}$$

$$V_\xi \quad = \quad \sum_{i=1}^{2} v_i |h_i|^2 + \sigma^2 - v_j |h_j|^2, \tag{13}$$

where the soft symbol $\hat{x}_i$ and the 'instantaneous' variance v_i are given by:

$$\hat{x}_i \quad = \quad \sum_{x \in \mathcal{A}} x \Pr^a(x_j = x), \tag{14}$$

$$v_i \quad = \quad \sum_{x \in \mathcal{A}} |x|^2 \Pr^a(x_j = x) - |\hat{x}_i|^2. \tag{15}$$

For the decoder of a binary code, the extrinsic non-binary symbol probability $\Pr^e(x_j)$ may be converted to the bit-based extrinsic LLR $\mathcal{L}^e_{MUD}(s_j = d^q_j), q \in [1, Q]$, where we have $Q = \log_2|\mathcal{A}|$ and $|\mathcal{A}|$ is the cardinality, i.e. the number of phases in the modulation alphabet $\mathcal{A}$. The extrinsic LLR of the resultant bit is thus given by:

$$\mathcal{L}^e_{MUD}(s_j = d^q_j) = \log_2 \frac{\sum_{x \in \mathcal{A}^+_q} \Pr^e(x_j = x) \Pr^a(x_j = x)}{\sum_{x \in \mathcal{A}^-_q} \Pr^e(x_j = x) \Pr^a(x_j = x)}, \tag{16}$$

where $\mathcal{A}^+_q$ and $\mathcal{A}^-_q$ denotes the two subsets of A hosting symbols with their qth bit being +1 and -1, respectively. It can be seen from Eq. (16) that in the derivation of the extrinsic information $\mathcal{L}^e_{MUD}(s_j = d^q_j)$, only the *a priori* symbol probability $\Pr^a(x_j = x)$ is needed, which is given by:

$$\Pr^a(x_j = x) = \prod_{q \in [1,Q]} \frac{1}{2} \left\{1 + x^q \tanh\left[\mathcal{L}^a_{MUD}(s_j = d^q_j)/2\right]\right\},$$

where $x^q \in \{\pm 1\}$ is the qth bit's polarity in symbol x. This corresponds to a bit-LLR to symbol-probability conversion, where the bit LLR $\mathcal{L}^a_{MUD}(s_j = d^q_j)$ is gleaned from the output of the SECCC-ID decoder block. It is then deinterleaved using π_1^{-1} of Fig 2 to generate $\mathcal{L}^a_{DEC}(c)$. The extrinsic LLRs of the codeword are denoted by $\mathcal{L}^e_{DEC}(c)$ at the output of the SECCC-ID decoder, which are fed back to the MUD of Fig 2. Then they are interleaved by π_1, thus completing the outer iteration between the SECCC-ID decoder and the MUD. Soft-Input Soft-Output (SISO) *Maximum A Posteriori Probability* (MAP) SECCC-ID decoder [22] first calculates the extrinsic LLR of the information bits, namely $L_e(b_1)$ and $L_e(b_2)$. Then they are appropriately interleaved to yield the *a priori* LLRs of the information bits, namely $L_a(b_1)$ and $L_a(b_2)$, as shown in Fig 3. Self-concatenated decoding proceeds, until a fixed number of iterations is reached. Apart from having inner self-concatenated iterations in the SECCC decoder, a fixed number of outer iterations exchange extrinsic information between the decoder and the MUD [23] in order to yield the decoded bits $\hat{b}_1$. A similar procedure is followed, when generating $\hat{b}_2$.

C. Phase II - Relay to Source Transmission

At the relay re-encoding of $\hat{b}_1$ and $\hat{b}_2$ is carried out using a rate-R SECCC encoder and the resultant QPSK modulated signals are $\hat{x}_1$ and $\hat{x}_2$. These are re-transmitted as x_R.

We hence focus our attention on source A, which received the signal $y_A = h_1 x_R + n_A$, where the transmitted signal x_R generated by the SPC scheme may be written as:

$$x_R = \rho \hat{x}_1 + (1-\rho)\hat{x}_2, \quad (17)$$

with ρ being the amplitude scaling factor used at the relay for SPC and we simply assume $\rho = 1/2$ in our paper. Node A receives $y_1 = h_1 * f(\hat{b}_1) + h_1 * f(\hat{b}_2) + n_1$, where the function $f(.)$ represents the SECCC encoding. Since A has its own *a priori* information of b_1, it will first construct $f(b_1)$ using the same encoding function at the relay's transmiter and subtracts the information from the received signal y_1 and then decodes B's information as seen in Fig 2. However, in general $\hat{b}_1 \neq b_1$, hence $f(\hat{b}_2) \neq f(b_2)$. Given perfect self-information of x_1 at source A, we could simply initilise the detection process by the *a priori* information provided for source A according to its self-information. Note that this *a priori* information generated by pefect self-information may not be equal to the actual *a priori* information of our real transmitted packet, which is $\hat{x}_1$, unless pefect reception is assumed at the relay, when we have $\hat{x}_1 = x_1$, which may be referred to as the *idealized* scenario. By contrast, in a *realistic* scenario we have $\hat{x}_1 \neq x_1$. The receiver at source B follows the same design methodology.

III. Performance Evaluation

A. Assumptions and Parameters

We have investigated the performance of our system for transmission over both AWGN and uncorrelated Rayleigh fading channels. The SECCC scheme of Fig 3 used $R_1 = 1/2$, $R_2 = 3/4$, $\nu = 3$ and Gray mapping, hence its overall rate [18] R=1/3 from Eq. 9. The bit-to-symbol mappers are QPSK mappers. Hence, we have $\eta = 0.67$ bit/s/Hz. The same encoder is used in our SPC-SECCC scheme, as depicted in Fig 2. Since there are two users, the normalized per-user transmission rate

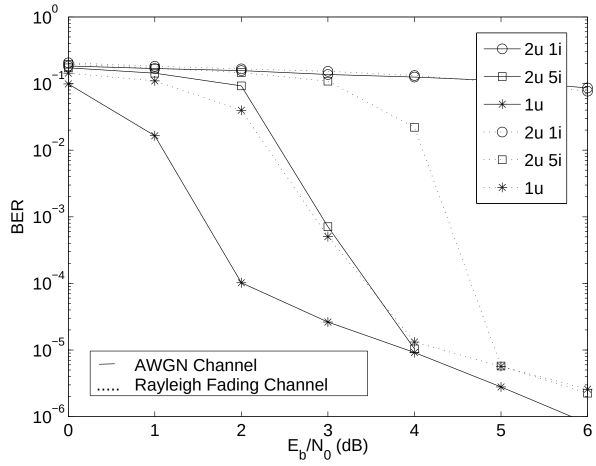

Fig. 4. BER versus $E_b/N_0 = [E_b/N_0]_r$ of Phase-I of SPC-SECCC scheme for transmission over both AWGN and uncorrelated Rayleigh fading channels.

of the two users exchanging information in two time slots will be $2/3$ bit/symbol. Similarly, the direct link based transmission between the two users each having a transmission rate of $2/3$ bit/symbol also exchanges information in two different time slots, as shown in Fig 1. The SPC-SECCC scheme imposes the additional complexity of the twin-stream detector, but does not cause any throughput reduction compared to an SECCC scheme. We considered an information block length of 1000 bits per frame and the number of SECCC decoding iterations was fixed to $I_{sd} = 10$.

B. Simulation Results

We consider a relay node located at the mid-point between the source/destination nodes A and B. According to Eqs 3 and 4, we have $G_{ar} = G_{br} = G_{ra} = G_{rb} = 4$, while $G_{ab} = 1$. Hence from Eq 7 we have, $[E_b/N_0]_r$ = $[E_b/N_0]_t$ + $10\log_{10}(G_{ar})$. The achievable Phase-I performance is characterized in Fig 4 for transmission over both AWGN and uncorrelated Rayleigh fading channels. The various scenarios considered were denoted by the legends: '2u 1i', '2u 5i', and '1u', where (i) represents the number of outer iterations between the SPC and SECCC decoders, as follows:

- 2u 1i - 2 users and 1 iteration;
- 2u 5i - 2 users and 5 iterations;
- 1u - 1 user.

Observe in Fig 4 that a marked improvement is exhibited by the BER curve both for AWGN and Rayleigh fading channels, when outer iterations are invoked. For the AWGN channel, represented by the solid line, at a BER of 10^{-4} the two-user scenario requires a 1.5 dB higher E_b/N_0 than the single-user case. Similar trends may be observed for Rayleigh fading channels, as indicated by the dashed lines in Fig 4.

The corresponding Phase-II performance is shown in Fig 5 for an AWGN scenario and in Fig 6 for a Rayleigh fading channel, respectively. The performance of the various scenarios is characterized by the four curves: '2u w 5i real', '2u wo 5i real', '2u w 1i', '1u dt', which represent the following scenarios:

- 2u w 5i real - with *a priori* information using $I_o = 5$ outer iterations for realistic error propagation at the relay;

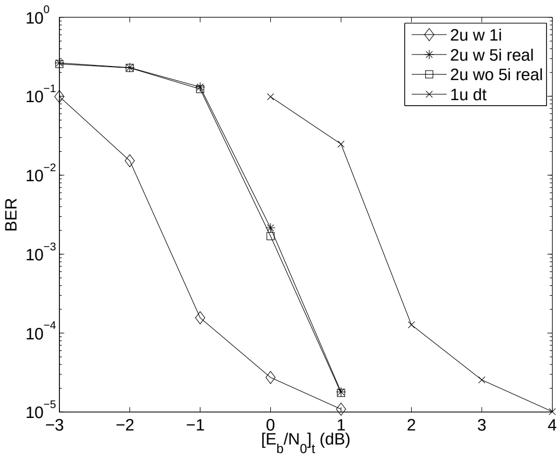

Fig. 5. BER versus $[E_b/N_0]_t$ of Phase-II of the SPC-SECCC scheme for transmission over an AWGN channel.

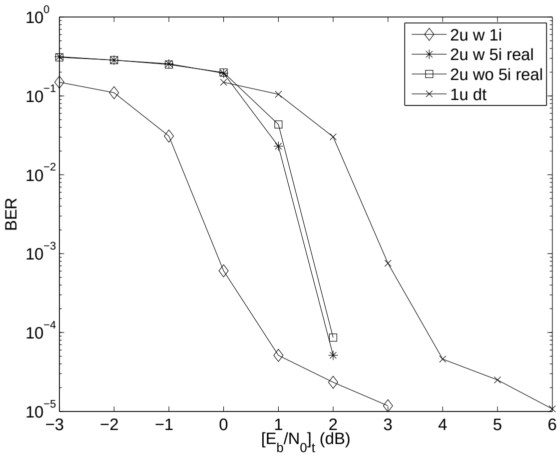

Fig. 6. BER versus $[E_b/N_0]_t$ of Phase-II of the SPC-SECCC scheme for transmission over an uncorrelated Rayleigh fading channel.

- 2u wo 5i real - without *a priori* information using $I_o = 5$ outer iterations for realistic error propagation at the relay;
- 2u w 1i - with *a priori* information using $I_o = 1$ outer iteration and neglecting error propagation at the relay;
- 1u dt - when we consider having only a direct link between node A and B employing SECCC schemes. Due to the higher geographical distance between the two nodes the attainable performance was degraded. The SPC-SECCC scheme performs better than the '1u dt' scheme, although they have the same throughput.

It was concluded that in the absence of errors at the relay, node A benefits from SPC. In Fig 5 the performance of the '2u w 1i' scenario was 3 dB better compared to '1u dt'. When assuming the presence of potential errors at the relay, node A still benefits from SPC and its performance is about 1 dB better than the single-user performance at a BER of 10^{-4} for the AWGN scenario and about 1.75 dB better for transmission over Rayleigh channels.

IV. Conclusions

In this paper we have proposed a bi-directional relaying-aided transmission scheme employing SPC and SECCC-ID. This enables us to reduce the number of time slots compared to both conventional relaying and NC, from four to two while keeping the normalized per-user throughput the same as that of two single users transmitting and receiving each other's information in two different time slots. Realistic transmission scenarios were considered, where the relay may encounter decision errors. The performance results gleaned from Fig 5 and 6 suggest that the SPC-SECCC system achieves a low BER even for realistic error-infested relaying. Our future work will concentrate on supporting an increased number of users under different channel conditions.

References

[1] P. Bergmans and T. Cover, "Cooperative broadcasting," *IEEE Transactions on Information Theory*, vol. 20, no. 3, pp. 317–324, May 1974.

[2] E. Larsson and B. Vojcic, "Cooperative transmit diversity based on superposition modulation," *IEEE Communications Letters*, vol. 9, pp. 778–780, Sept. 2005.

[3] X. Ma and L. Ping, "Coded modulation using superimposed binary codes," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3331–3343, Dec. 2004.

[4] C. Hausl and J. Hagenauer, "Iterative network and channel decoding for the two-way relay channel," in *Proceedings of IEEE ICC '06*, Istanbul, Turkey, June11-15, 2006, pp. 1568–1573.

[5] C. Hausl and P. Dupraz, "Joint network-channel coding for the multiple-access relay channel," in *Proceedings of IEEE Sensor and Ad Hoc Communications and Networks (SECON) '06*, Hyatt Regency, USA, Sept.25-28, 2006, pp. 817–822.

[6] N. Fawaz, D. Gesbert, and M. Debbah, "When network coding and dirty paper coding meet in a cooperative ad hoc network," *IEEE Transactions on Wireless Communications*, vol. 7, pp. 1862–1867, May 2008.

[7] L. Xiao, T. Fuja, J. Kliewer, and D. Costello, "A network coding approach to cooperative diversity," *IEEE Transactions on Information Theory*, vol. 53, pp. 3714–3722, Oct. 2007.

[8] R. Zhang and L. Hanzo, "Superposition-Coding Aided Multiplexed Hybrid ARQ Scheme for Improved End-to-End Transmission Efficiency," *IEEE Transactions on Vehicular Technology*. [Online]. Available: http://eprints.ecs.soton.ac.uk/17230/

[9] G. Forney, *Concatenated Codes*. MIT Press, 1966.

[10] C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: Turbo codes," *IEEE Transactions on Communications*, vol. 44, no. 10, pp. 1261–1271, October 1996.

[11] S. Benedetto, D. Divsalar, G. Montorsi, and F. Pollara, "Serial concatenated trellis coded modulation with iterative decoding," in *IEEE International Symposium on Information Theory*, Ulm, June/July 1997, p. 8.

[12] ——, "Self-concatenated trellis coded modulation with self-iterative decoding," in *IEEE Global Telecommunications Conference*, vol. 1, Sydney, NSW, Australia, 1998, pp. 585–591.

[13] H, Ochiai, P. Mitran and V. Tarokh, "Design and analysis of collaborative diversity protocols for wireless sensor networks," in *Proceedings of IEEE VTC Fall*, Los Angeles, USA, 26-29 September 2004, pp. 4645 – 4649.

[14] Z. Shengli, S.-C. Liew and P.P.K. Lam, "Physical Layer Network Coding," *Computer Science - Information Theory*, April 2007.

[15] S.X. Ng, M.F.U. Butt and L. Hanzo, "On the Union Bounds of Self-Concatenated Convolutional Codes," *IEEE Signal Processing Letters*, vol. 16, no. 9, pp. 754–757, September 2009.

[16] G. Caire, G. Taricco, and E. Biglieri, "Bit-interleaved coded modulation," *IEEE Transactions on Information Theory*, vol. 44, pp. 927–946, May 1998.

[17] L. Hanzo, T. Liew, and B. Yeap, *Turbo Coding, Turbo Equalisation and Space-Time Coding for Transmission over Fading Channels*. Wiley-IEEE Press, 2002.

[18] J. Hagenauer, "Rate-compatible punctured convolutional codes (RCPC codes) and their applications," *IEEE Transactions on Communications*, vol. 36, no. 4, pp. 389–400, Apr. 1988.

[19] M. F. U. Butt, R. A. Riaz, S. X. Ng, and L. Hanzo, "Near-capacity iteratively decoded binary self-concatenated code design using EXIT charts," in *IEEE Global Communications Conference, GLOBECOM '08*, New Orleans, USA, Nov./Dec. 2008.

[20] L. Hanzo and T. Keller, *OFDM and MC-CDMA: A Primer*. Wiley-IEEE Press, 2006.

[21] K. Wu and L. Ping, "Multilayer turbo space-time codes," *IEEE Transactions on Communications*, vol. 9, pp. 55–57, Jan. 2005.

[22] S. Benedetto, D. Divsalar, G. Motorsi, and F. Pollara, "A soft-input soft-output APP module for iterative decoding of concatenated codes," *IEEE Communication Letters*, pp. 22–24, 1997.

[23] L. Hanzo, O. Alamri, M. El-Hajjar and N. Wu, *Near-Capacity Multi-functional MIMO Systems*. IEEE Press - John Wiley, April 2009.