Turbo-Coded Cooperative Diversity for CDMA System

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***Abstract*—In this contribution we study a range of turbo-coded cooperative diversity schemes in the context of a Code Division Multiple Access (CDMA) system. The cooperative diversity architectures considered are the Amplify-and-Forward (AF), Decode-and-Forward (DF) and Soft Relay (SR) schemes. We derived the soft-decision metric for the turbo decoder based on the Minimum Mean Square Error (MMSE) detection when considering the AF, DF and SR schemes. The effect of the relay position on the cooperative diversity system performance was studied. It was found that the AF-based turbo-coded CDMA system is the most attractive in terms of performance and complexity.**

***Index Terms*—Code Division Multiple Access, Amplify-and-Forward, Decode-and-Forward, Soft Relay, Minimum Mean Square Error, Turbo Codes.**

I. INTRODUCTION

In wireless communication systems, the transmitted signals are distorted by various channel-induced attenuation such as multi-path fading, shadow fading and Multiple-User Interference (MUI). Diversity-based schemes are very effective in mitigating the channel fading. More specifically, temporal diversity can be exploited via various channel codes, such as turbo codes [1], [2], while frequency diversity can be obtained using channel equaliser and spatial diversity can be attained with the aid of multiple transmit/receive antennas [2]. Multiple-Transmitter Multiple-Receiver (MTMR) scheme is capable of achieving diversity gain without bandwidth expansion but at the cost of a more complex hardware complexity for implementing multiple transmit/receive antennas.

Cooperative diversity [3]–[7] scheme is another attractive spatial diversity method for obtaining diversity via cooperative communication between mobile users, where each user terminal employs only one transmit/receive antenna. Hence, multiple transmit antennas in the MTMR scheme is replaced by multiple cooperative users, or relays, in the cooperative diversity scheme. Amplify-and-Forward (AF) and Decode-and-Forward (DF) [4] are the frequently used cooperative diversity methods. The relay of the AF scheme amplifies the received signal before forwarding it to the destination terminal. By contrast, the relay of the DF scheme makes a hard decision of the received signal and forward it to the destination. There are other cooperative schemes such as the Soft Relay (SR) method where the relay computes the soft information based on the received signal and send the soft information to the destination. The soft information can be in the form of probability [8] or soft-estimation [7].

In this contribution, we will study the performance of a turbo-coded CDMA system when employing cooperative diversity schemes of AF, DF and soft-estimation based SR. The Minimum Mean Square Error (MMSE) based Multi-User Detection (MUD) is invoked at the destination. We also quantify the system performance by taking into consideration the effect of path loss in the communication channels. The rest of the paper is organised as follows. Section II introduces the system model and detection algorithm. Section III introduces the path loss incurred in cooperative communication. In Section IV we present the simulation results and our conclusion is offered in Section V.

II. SYSTEM MODEL

The block diagram of a turbo-coded cooperative diversity scheme employing AF algorithm in a CDMA system is shown in Figure 1. A half-rate memory-two turbo code having a generator polynomial of $(7,5)_8$ was used and the Binary-Phase Shift Keying (BPSK) modulation was employed. There are K number of CDMA users and each transmission frame consists of N_b number of turbo-coded bits. The data bits of each user $\{b_k\}$, for $k \in \{1,2,\ldots,K\}$, are first turbo-coded and BPSK-modulated before they are multiplied with its CDMA codeword of N-chips. Then, they are transmitted to both the relay and destination during the first transmission period of N_b bits (or $N \times N_b$ chips). The $[N \times 1]$-dimensional received signal vector for each N-chip period at the destination and that at the relay can be written as:

$$Y_{d,1} = SH_{s,d}X + N_{s,d} \quad (1)$$

and

$$Y_r = SH_{s,r}X + N_{s,r} \quad (2)$$

respectively, where X is the $[K \times 1]$-dimensional transmitted symbol vector by K users. The notations $H_{s,d}$ and $H_{s,r}$ represents the $[K \times K]$-dimensional diagonal matrix of the complex-valued Rayleigh fading coefficients from source to destination

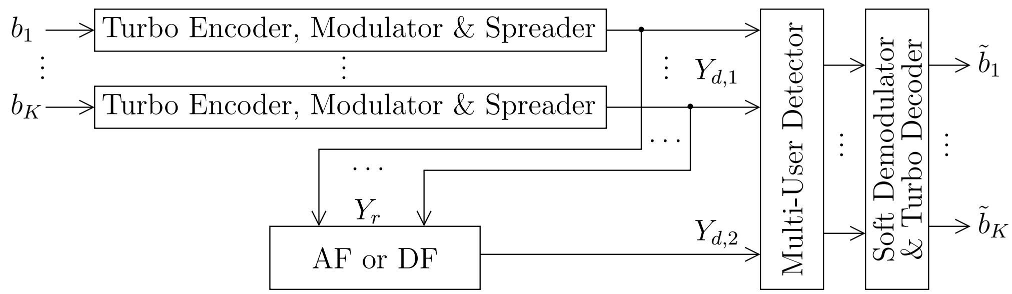

Figure 1. Block diagram of the turbo-coded cooperative diversity scheme employing AF or DF algorithm in a CDMA system.

and from source to relay respectively. S denotes the $[N \times K]$-dimensional spreading sequence where its kth column is the spreading sequences of the kth user, for $k \in \{1, 2, \ldots, K\}$. The notations $N_{s,d}$ and $N_{s,r}$ denotes the $[N \times 1]$-dimensional AWGN vector from source to destination and from source to relay, respectively. During the second transmission period, the relay will amplify and forward its received signal Y_r to the destination. Note that, all user terminals do not transmit during the second transmission period. The $[N \times 1]$-dimensional received signal vector for each N-chip period at the destination during the second transmission period is given by:

$$Y_{d,2} = h_{r,d} \beta Y_r + N_{r,d} \tag{3}$$

where $h_{r,d}$ is Rayleigh fading coefficient from relay to destination, β is the amplification factor and $N_{r,d}$ denotes the $[N \times 1]$-dimensional AWGN vector from relay to destination. Let us now look at the detection at the relay and destination, where an MMSE-based MUD detection is employed at the destination.

A. Source to Destination

During the first transmission period, the received signal $Y_{d,1}$ at the destination is given by Eq (1). The signals of all users can be detected using the following MMSE-based MUD:

$$\overline{Z}_{d,1} = \overline{R}^{-1} S^H Y_{d,1} \tag{4}$$

$$= \overline{R}^{-1} S^H S H_{s,d} X + \overline{R}^{-1} S^H N_{s,d} \tag{5}$$

where $\overline{R} = S^H S + 2\sigma^2 I$, $\sigma^2 = N_0/2$ is the noise variance per dimension and I is the $[K \times K]$-dimensional identity matrix. We assume that the channel fading coefficient is constant during each bit period (or N chip periods). The resultant signal $\overline{Z}_{d,1}$ is multiplied with the conjugate transpose of the channel matrix to yield:

$$\widetilde{Z}_{d,1} = H_{s,d}^H \overline{Z}_{d,1} \tag{6}$$

$$= |H_{s,d}|^2 \overline{R}^{-1} S^H S X + H_{s,d}^H \overline{R}^{-1} S^H N_{s,d} \tag{7}$$

where the first row of $\widetilde{Z}_{d,1}$ is the MUD output for user 1. In order to provide a soft-decision metric for the turbo decoder, we need to determine the equivalent noise variance from Equation. (7) for each user. The matrix $\overline{R}^{-1} S^H$ is $[K \times N]$-dimensional where its kth row is related to user k. The equivalent noise variance for each user can be calculated. For example, the equivalent noise variance for user 1 can be computed as:

$$\overline{N}_{o,1} = |h_{s,d,1}|^2 ||M_1||^2 N_0 \tag{8}$$

where M_1 is the first row of $\overline{R}^{-1} S^H$ and $h_{s,d,1}$ is the channel fading coefficient from user 1 at the source to the destination. The Probability Density Function (PDF) of receiving $\widetilde{Z}_{d,1}$ conditioned on a transmitted signal $x_1 \in \{+1, -1\}$ from user 1 can be computed based on Equations. (7) and (8) as:

$$P(\widetilde{Z}_{d,1}|x_1) = \frac{1}{\pi \overline{N}_{o,1}} \exp \left[\frac{-\left| \widetilde{Z}_{d,1} - a|h_{s,d,1}|^2 x_1 \right|^2}{\overline{N}_{o,1}} \right] \tag{9}$$

where $a = J_{1,1}$ is the $(1, 1)$ element of the matrix $J = \overline{R}^{-1} S^H S$. It can be shown that the Log-Likelihood Ratio (LLR) of the turbo-coded bit x_1 conditioned on receiving $\widetilde{Z}_{d,1}$ can be computed from:

$$L(x_1|\widetilde{Z}_{d,1}) = 4a|h_{s,d,1}|^2 \frac{E_b}{\overline{N}_{o,1}} R \cdot \widetilde{Z}_{d,1} \tag{10}$$

$$= 4 \frac{a}{||M_1||^2} \frac{E_b}{N_0} R \cdot \widetilde{Z}_{d,1} \tag{11}$$

$$= L_{c,1} \cdot y_1 \tag{12}$$

where $L_{c,1} = 4\frac{a}{||M_1||^2} \frac{E_b}{N_0} R$ and $y_1 = \widetilde{Z}_{d,1}$. The coding rate of the system is given by $R = R_c \times R_r = 0.25$, where $R_c = 0.5$ is the coding rate of the turbo encoder and $R_r = 0.5$ is the rate of the cooperative diversity scheme which utilizes two bit periods for transmitting a single bit. The notation E_b/N_0 represents the signal-to-noise ratio per information bit.

B. Source to Relay

At the relay of the AF scheme, the received signal as given by Equation. (2) is amplified by an amplification factor of β before it is forwarded to the destination during the second transmission period as $\overline{Y}_{s,r} = \beta Y_r$. The amplification factor can be derived as [9]:

$$\beta = \sqrt{\frac{\xi}{C\xi + N_0}} \tag{13}$$

where ξ is the transmission signal energy, $C = |h_{s,r,1}|^2 + \ldots + |h_{s,r,k}|^2 + \ldots + |h_{s,r,K}|^2$ and $h_{s,r,k}$ is the complex-valued Rayleigh fading coefficient for user k, for $k \in \{1, 2, \ldots, K\}$.

C. Relay to Destination

During the second transmission period, the signals received at the destination from the relay is given by Equation. (3). The signals of all users can be detected based on the MMSE MUD as:

$$\begin{aligned}\overline{Z}_{d,2} &= \overline{R}^{-1} S^H Y_{d,2} && (14)\\ &= \overline{R}^{-1} S^H S h_{r,d} \beta H_{s,r} X + \\ &\quad \overline{R}^{-1} S^H h_{r,d} \beta N_{s,r} + \overline{R}^{-1} S^H N_{r,d} && (15)\end{aligned}$$

where $\overline{R} = S^H S + 2\sigma^2 I$ as in Equation. (5). Similar to Equation. (7), the MUD output for user 1 can be computed as:

$$\begin{aligned}\widetilde{Z}_{d,2} &= h^*_{r,d} \overline{Z}_{d,2} \ , && (16)\\ &= |h_{r,d}|^2 \overline{R}^{-1} S^H \beta S H_{s,r} X + \\ &\quad |h_{r,d}|^2 \overline{R}^{-1} S^H \beta N_{s,r} + h^*_{r,d} \overline{R}^{-1} S^H N_{r,d} && (17)\end{aligned}$$

Again, let M_1 be the first row of the matrix $\overline{R}^{-1} S^H$ and the equivalent noise variance can be calculated for user 1 as:

$$\overline{N}_{o,r,1} = [|h_{r,d}|^4 \beta^2 ||M_1||^2 + |h_{r,d}|^2 ||M_1||^2] N_0 \qquad (18)$$

The PDF of receiving the first row of $\widetilde{Z}_{d,2}$ conditioned on a transmitted signal $x_1 \in \{+1, -1\}$ from user 1 can be computed based on Equations. (17) and (18) as:

$$P(\widetilde{Z}_{d,2}|x_1) = \frac{1}{\pi \overline{N}_{o,r,1}} \exp\left[\frac{-\left|\widetilde{Z}_{d,2} - a|h_{r,d,1}|^2 \beta h_{s,r,1} x_1\right|^2}{\overline{N}_{o,r,1}}\right] \qquad (19)$$

where again $a = J_{1,1}$ is the $(1,1)$ element of the matrix $J = \overline{R}^{-1} S^H S$. It can be shown that the LLR of the turbo-coded bit x_1 conditioned on receiving $\widetilde{Z}_{d,2}$ can be computed as:

$$\begin{aligned}L(x_1|\widetilde{Z}_{d,2}) &= \frac{4a\beta}{(|h_{r,d,1}|^2 \beta^2 + 1)||M_1||^2} \frac{E_b}{N_0} R \cdot \widetilde{Z}_{d,2}\\ &= L_{c,2} \cdot y_2 && (20)\end{aligned}$$

where $L_{c,2} = \frac{4a\beta}{(|h_{r,d,1}|^2\beta^2+1)||M_1||^2} \frac{E_b}{N_0} R$ and $y_2 = \widetilde{Z}_{d,2}$.

Finally, the joint PDF of transmitting x_1 given that $\widetilde{Z}_{d,1}$ and $\widetilde{Z}_{d,2}$ are received can be computed as:

$$P(x_1|\widetilde{Z}_{d,1}, \widetilde{Z}_{d,2}) = \frac{P(\widetilde{Z}_{d,1}, \widetilde{Z}_{d,2}|x_1) \cdot P(x_1)}{P(\widetilde{Z}_{d,1}, \widetilde{Z}_{d,2})} \qquad (21)$$

where $P(\widetilde{Z}_{d,1}, \widetilde{Z}_{d,2})$ is a constant that can be ignored and $P(x_1)$ is the *a priori* probability of x_1 which is given by $P(x_1) = 0.5$ when there is no feedback from the channel decoder. Furthermore, since $\widetilde{Z}_{d,1}$ and $\widetilde{Z}_{d,2}$ were received during two different timeslots, they can be rendered independent. Hence, Equation. (21) may be rewritten as:

$$P(x_1|\widetilde{Z}_{d,1}, \widetilde{Z}_{d,2}) = P(\widetilde{Z}_{d,1}|x_1) \cdot P(\widetilde{Z}_{d,2}|x_1) \cdot P(x_1) \ , \qquad (22)$$

which can be used as the soft-decision metric to the turbo decoder. We may also compute the LLR of x_1 conditioned on $\widetilde{Z}_{d,1}$ and $\widetilde{Z}_{d,2}$ as:

$$L(x_1|\widetilde{Z}_{d,1}, \widetilde{Z}_{d,2}) = L(x_1|\widetilde{Z}_{d,1}) + L(x_1|\widetilde{Z}_{d,2}) + L(x_1) \qquad (23)$$

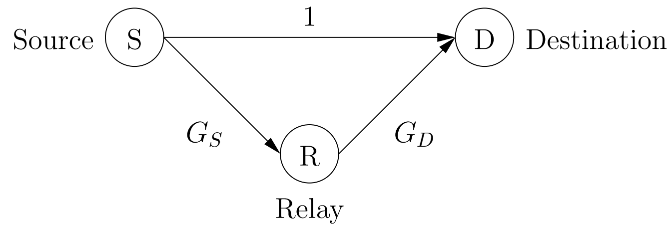

Figure 2. Path loss effect in a cooperative communication.

where $L(x_1)$ is the *a priori* LLR of x_1 which can be obtained from the turbo decoder feedback.

III. Path Loss

In this section, the effect of relay geometry is considered as shown in Figure 2. Let $d_{A,B}$ denotes the Euclidean distance between nodes A and B. For simplicity, the path loss between these nodes is modelled as [10]:

$$PL(A, B) = \frac{K_P}{d^{\alpha}_{A,B}} \qquad (24)$$

where K_p that depends on the environment and α is the path loss exponent. Assuming a unit path loss between source and destination, the received energy at the relay is related to the received energy at the destination according to [10]:

$$\begin{aligned}E_{S,R} &= \frac{PL(S,R)}{PL(S,D)} E_{S,D}\\ &= \left(\frac{d_{S,D}}{d_{S,R}}\right)^{\alpha} E_{S,D}\\ &= G_S E_{S,D} && (25)\end{aligned}$$

where G_S is the geometrical gain while the notations S, R and D are used to denote the source, relay and destination terminals, respectively. The destination's received energy from the relay node is related to that from the source node according to [10]:

$$\begin{aligned}E_{R,D} &= \left(\frac{d_{S,D}}{d_{R,D}}\right)^{\alpha} E_{S,D}\\ &= G_D E_{S,D} && (26)\end{aligned}$$

From the above expression, we can see that if the relay transmits its signal with Δ_R times the average power of the source, then the destination node receives the relay's signal with a gain of $\Delta_R G_D$ compared to that of the source to destination channel link.

In this study, we assume that the relay node is located on a direct path between the source and the destination nodes, i.e.:

$$d_{S,D} = d_{S,R} + d_{R,D} \ . \qquad (27)$$

Hence, from Equations (25), (26) and (27), we have:

$$\begin{aligned}1 &= \frac{1}{\sqrt[\alpha]{G_S}} + \frac{1}{\sqrt[\alpha]{G_D}}\\ G_D &= \left(\frac{1}{1 - 1/\sqrt[\alpha]{G_S}}\right)^{\alpha} \ , && (28)\end{aligned}$$

where we employ a sub-urban path loss model [11] having a path loss exponent of $\alpha = 4$.

For AF based cooperation, the two received signals concerning bit x_1 at the destination can be expressed as:

$$Y_{d,1} = SH_{s,d}X + N_{s,d} \quad (29)$$

and

$$\begin{aligned} Y_{d,2} &= \sqrt{G_D} h_{r,d} (\beta Y_r) + N_{r,d} \\ &= \sqrt{G_D} h_{r,d} \beta \sqrt{G_S} SH_{s,r} X + \\ & \quad \sqrt{G_D} h_{r,d} \beta N_{s,r} + N_{r,d} \end{aligned} \quad (30)$$

where β can be expressed as:

$$\beta = \sqrt{\frac{\xi}{G_S |H_{s,r}|^2 \xi + N_0}} \quad (31)$$

For DF based cooperation where each user has one independent relay, the two received signals concerning bit x_1 at the destination can be expressed as:

$$Y_{d,1} = SH_{s,d}X + N_{s,d} \quad (32)$$

and

$$Y_{d,2} = \sqrt{G_D} SH_{r,d} X_r + N_{r,d} \quad (33)$$

where X_r is the $[K \times 1]$-dimensional symbol vector transmitted from the K number of relays and $H_{r,d}$ is the $[K \times K]$-dimensional diagonal matrix of the complex-valued Rayleigh fading coefficients from relays to destination.

IV. Results and Discussion

We have also use MMSE detection for both the DF and the soft-estimation based SR using a similar approach as described in Section II for AF scheme. However, the details of the MMSE detection for the DF and SR scheme were not presented due to the lack of space. We consider a block length of 1000 informations bits in each transmission frame and the CDMA system supports $K = 7$ users where each spreading sequence has $N = 7$ chips. The transmission is over uncorrelated flat Rayleigh fading channels. Eight turbo iterations are used at the destination (for all schemes) and relay terminals (for the DF and SR schemes).

Figures 3, 4 and 5 compare the Bit Error Ratio (BER) versus relay position (or geometrical gain G_S) performance of the AF, DF and SR schemes when consider E_b/N_0 values of 2 dB, 4 dB and 6 dB, respectively. At a given G_S value, the corresponding G_D value can be computed from Equation (28). As seen from Figures 3, 4 and 5, the SR scheme requires a sufficiently high E_b/N_0 value before it is capable of providing a good performance. The SR scheme is also less sensitive to the relay position. Note that at a higher G_S value, the relay is positioned nearer to the source. When the three terminals are having the same distance from each other, the G_S value is given by $G_S = 4$. As shown in Figures 3, 4 and 5, the DF scheme exhibits a poor performance when the relay is very close to the source ($G_S \geq 36$) or very near the destination ($G_S \leq 1$). However, the AF scheme is capable of providing a good BER performance (BER$< 10^{-4}$) for a wide range of relay positions ($1 \leq G_S \leq 36$) when the E_b/N_0 value is sufficiently high ($E_b/N_0 = 6$ dB). It is interesting to find that in the turbo-coded cooperative system, the AF scheme outperform the DF scheme in most scenarios, especially at high E_b/N_0 values. This is attractive, since the relay for the AF scheme has a lower complexity compared to that of the DF scheme. We also found the similar performance trend when no channel coding is used although the uncoded schemes require a higher E_b/N_0 value in order to attain the same BER.

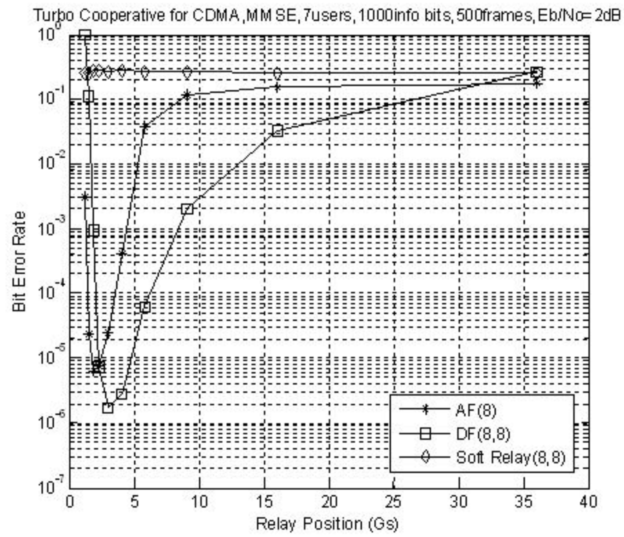

Figure 3. Comparisons of AF, DF and SR algorithms with respect to relay position, eight turbo iterations at the destination and relay, $E_b/N_0 = 2$ dB.

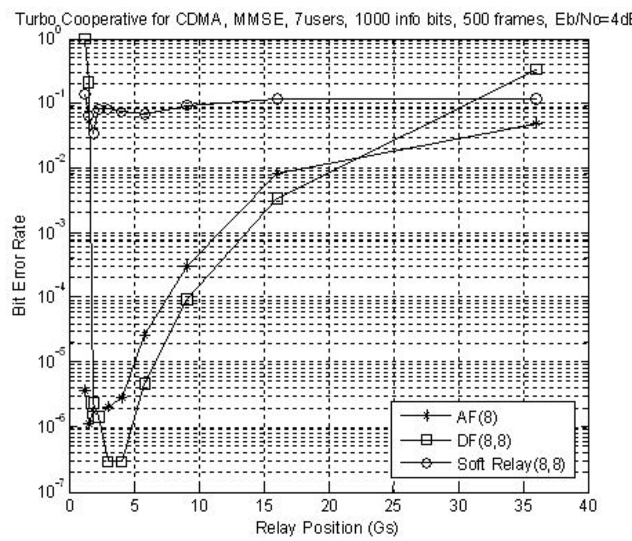

Figure 4. Comparisons of AF, DF and SR algorithms with respect to relay position, eight turbo iterations at the destination and relay, $E_b/N_0 = 4$ dB.

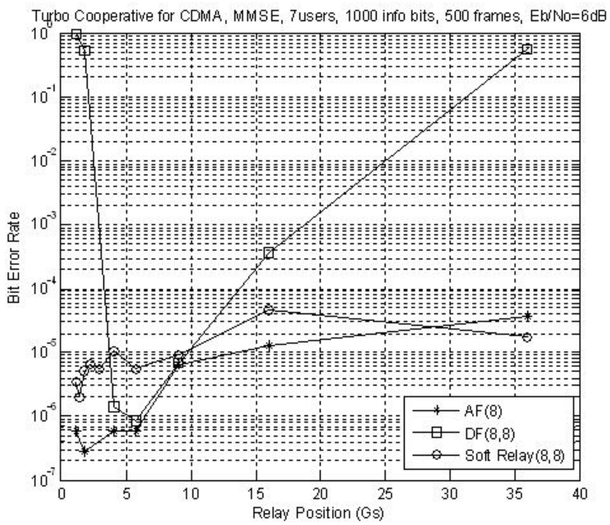

Figure 5. Comparisons of AF, DF and SR algorithms with respect to relay position, eight turbo iterations at the destination and relay, $E_b/N_0 = 6$ dB.

V. Conclusions

We have studied various turbo-coded cooperative diversity schemes when communicating in a CDMA system. The SR-based scheme is less sensitive to the relay position but requires a higher E_b/N_0 value in order to exhibit a good BER performance. The AF-based scheme is the most efficient scheme since it is capable of providing good BER performance at the cost of a simple relay. The turbo-coded cooperative diversity schemes are capable of achieving good BER performance with the aid of the proposed soft-decision metric based MMSE detection. We will consider non-coherent cooperative diversity schemes in our future works.

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